

Math 2370 – Fall 2006
Practice Problems IX

Problem 1: Suppose that $T \in L(V, V)$ where V is finite dimensional, is such that every subspace with dimension $\dim V - 1$ is invariant under T . Show that T is a scalar multiple of the identity map.

Problem 2: Recall the theorem (proved in the class) stating that λ is an eigenvalue of $A \in \mathbb{C}^{n \times n}$ if and only if $p(\lambda)$ is an eigenvalue of $p(A)$. Show that the theorem need hold if $\lambda \in \mathbb{R}$, $A \in \mathbb{R}^{n \times n}$ and p is defined over \mathbb{R} .

Problem 3: (a) Give an example of an invertible matrix with 0's on the diagonal.
(b) Give an example of a non-invertible matrix with all diagonal numbers non-zero.

Problem 4: Let $P \in L(V, V)$ be a projection (i.e., $P^2 = P$). Show that $V = R_P \oplus N_P$.

Problem 5: Find at least one eigenvector and eigenvalue of the real cyclic matrix

$$\begin{bmatrix} a_1 & a_2 & a_3 & \cdots & a_n \\ a_n & a_1 & a_2 & \cdots & a_{n-1} \\ a_{n-1} & a_n & a_1 & \cdots & a_{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_2 & a_3 & a_4 & \cdots & a_1 \end{bmatrix}$$

Problem 6: Show that the characteristic polynomial of the matrix A is $s^2(s-1)^2$ and that it is also its minimal polynomial:

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ -2 & -2 & 2 & 1 \\ 1 & 1 & -1 & 0 \end{bmatrix}$$

Problem 7: Let N be a 2×2 nonzero complex matrix such that $N^2 = 0$. Show that N is similar to $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.