Math 2370 – Fall 2006
Practice Problems XI
Due Thursday Nov 9 as HOMEWORK

Problem 1: What is the minimal polynomial of
(a) a projection (i.e., linear map *P* that obeys $P^2 = P$) ?
(b) an involution (i.e., linear map *U* that obeys $U^2 = I$)?
(c) a map *T* on P_n such that $T(p(t)) = p(t+1)$?

Problem 2: Under what conditions on the complex numbers $a_1, a_2, ..., a_n$ is the following matrix diagonalizable over $\mathbb{C}$?

$$\begin{bmatrix} 0 & \cdots & 0 & a_1 \\ 0 & \cdots & a_2 & 0 \\ \vdots & ⋰ & \vdots & \vdots \\ a_n & \cdots & 0 & 0 \end{bmatrix}$$

Problem 3: Which of these matrices is diagonalizable over $\mathbb{C}$? Which over $\mathbb{R}$?

$$\text{(a)} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad \text{(b)} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \text{(c)} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Problem 4: Find the Jordan form of $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

Problem 5: Let *A* and *B* be $n \times n$ complex matrices such that $A^n = B^n = 0$ and $A^{n-1} \neq 0 \neq B^{n-1}$. Show that *A* and *B* are similar.

Problem 6: Suppose that $T \in L(V,V)$ has a cyclic vector (i.e., there is a vector $x \in V$ such that $x, Tx, T^2x, ..., T^{n-1}x$ is a basis of *V*). Show that if $U \in L(V,V)$ and $UT = TU$ then U is a polynomial in *T*.

Problem 7: Suppose that $T \in L(V,V)$ and $\dim R_T = 1$. Show that *T* is either diagonalizable or nilpotent but not both.

Problem 8: Suppose that A is 2x2 real matrix with eigenvalues $a \pm ib$. Show that A is similar to $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$.

Problem 9: List all possible Jordan forms for a 5x5 complex matrix with characteristic polynomial $(s-2)^3(s+1)^2$. For each such form give its minimal polynomial.

Problem 10: Let A and B be linear operators on a finite-dimensional complex linear space V. Let p be any polynomial such that $p(AB) = 0$.

(a) Show that if $q(s) = sp(s)$, then $q(BA) = 0$

(b) Use the result of (a) to show that the minimal polynomials m_{AB} and m_{BA} obey either $m_{AB}(s) = m_{BA}(s)$ or $m_{AB}(s) = sm_{BA}(s)$.