Homework 2

1. Consider the equations from class:

$$
\begin{aligned}
r_1' &= r_1(1-r_1^2) + \mu(r_2\cos\phi - r_1) - \nu r_2 \sin\phi \\
r_2' &= r_2(1-r_2^2) + \mu(r_1\cos\phi - r_2) + \nu r_2 \sin\phi \\
\phi' &= q(r_1^2 - r_2^2) - \mu\left(\frac{r_2}{r_1} + \frac{r_1}{r_2}\right) + \nu\left(\frac{r_1}{r_2} - \frac{r_2}{r_1}\right)\cos\phi
\end{aligned}
$$

   There are simple solutions to this, $(r_1, r_2, \phi) = (1, 1, 0)$ and (ρ, ρ, π) where $\rho^2 = 1 - \mu$. The latter is called the anti-phase solution. Compute their stability. Note that you will get a 3×3 matrix that has the form

$$
M = \begin{pmatrix} a & b & c \\ b & a & -c \\ d & e & f \end{pmatrix}
$$

   If you let

$$
P = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

   Then $P^{-1}MP$ will be a block diagonal matrix and you can pick off one eigenvalue immediately. Apply the determinant and trace rule to the remaining 2×2 block to figure out stability. (Recall that eigenvalues of a 2×2 matrix have negatibe real parts if the determinant is positive and the trace negative.) If you get stuck look at the 1990 paper by Aronson, Ermentrout, and Kopell, section 5.

2. Consider the above system where $\nu = 1, q = 2$. For what values of μ is synchrony stable? How about the antiphase solution? Numerically solve the ODEs for $\mu = 0.5, 0.3$ and describe what you see. You should not start at exactly the simple solutions.

3. Numerically solve the coupled Brusselator equations

$$
\begin{aligned}
x_j' &= A - (B+1)x_j + x_j^2 y_j + D_x(x_k - x_j) \\
y_j' &= B x_j - x_j^2 y_j + D_y(y_k - y_j)
\end{aligned}
$$

   for $D_x = 0.05, A = 1, B = 2.5$ and $D_y = 0.2, 0.8, 1.2$ try several initial conditions to make sure you have found all the stable dynamics

4. Do exercises 1,3,4 in the PDF I gave you for chapter 8 of my book.

5. Consider the radial isochron clock in the figure. Use trig to compute ϕ_{new} as a function of ϕ where $0 < \alpha < 1$. This is called the phase transition curve. The phase resetting curve is

$$
\phi_{new}(\phi) - \phi = G(\phi)
$$

Compute the following limit

$$\lim_{\alpha \to 0} \frac{G(\phi)}{\alpha}$$

which is the infinitesimal phase-resetting curve.