Homework # 5

1. Problems 5,6,9 chapter 10 (page 322-323)

2. Consider the extended Wilson-Cowan model:

\[
\begin{align*}
\tau_{11} z_{11}' &= -z_{11} + f_1(g_{11} z_{11} + g_{12} z_{12}) \\
\tau_{21} z_{21}' &= -z_{21} + f_1(g_{11} z_{11} + g_{12} z_{12}) \\
\tau_{12} z_{12}' &= -z_{12} + f_2(g_{21} z_{21} + g_{22} z_{22}) \\
\tau_{22} z_{22}' &= -z_{22} + f_2(g_{21} z_{21} + g_{22} z_{22})
\end{align*}
\]

Prove that if \( \tau_{11} = \tau_{21} \) and \( \tau_{12} = \tau_{22} \) that all solutions to this ODE satisfy:

\[
\lim_{t \to \infty} |z_{1j}(t) - z_{2j}(t)| = 0, \quad j = 1, 2
\]

and thus, they reduce to the WC equations. By reversing the derivation in class, show that if \( \tau_{11} = \tau_{12} \) and \( \tau_{21} = \tau_{22} \), show that the resulting model reduces to two equations (The Hopfield model).

3. Consider the two models for a scalar neural network with second and third order synapses:

\[
\begin{align*}
u'' + au' + bu &= bf(gu + I), \quad (A) \\
u'' + au'' + bu' + cu &= cf(gu + I), \quad (B)
\end{align*}
\]

where \( a, b, c \) are all positive, and \( f'(y) \) is monotone increasing (e.g. \( f(u) = 1/(1 + \exp(-u)) \)) Can Model (A) undergo any Hopf Bifurcations to oscillations? How about Model (B) (Hint: use the Routh-Hurwitz criteria for these.) if your answer is yes to any of them, simulate an example. What is the sign of \( g \) in order to get oscillations. Prove that if \( g < 0 \), then each model has exactly one equilibrium point.

4. Problems 15,18 page 364-365

5. Page 399-401 number 1, 6,7,10