Homework

In this exercise, you will look at the Morris-Lecar equations and study their dynamics. The equations are:

\[
\begin{align*}
\frac{dv}{dt} &= \left( I - gca \cdot \minf(V) \cdot (V - Vca) - gk \cdot w \cdot (V - VK) - gl \cdot (V - Vl) \right) / c \\
\frac{dw}{dt} &= \phi \cdot \left( \winf(V) - w \right) / \tauw(V) \\
\minf(v) &= 0.5 \cdot \left( 1 + \tanh((v - va) / vb) \right) \\
\winf(v) &= 0.5 \cdot \left( 1 + \tanh((v - vc) / vd) \right) \\
\tauw(v) &= 1 / \cosh((v - vc) / (2 \cdot vd))
\end{align*}
\]

param \( v_k = -84, v_l = -60, vca = 120 \)
param \( i = 0, gk = 8, gl = 2, c = 20 \)
param \( va = -1.2, vb = 18 \)
param \( vc = 2, vd = 30, \phi = 0.04, gca = 4.4 \)

1. Plot the steady-state I-V curve by setting \( w = w_\infty(V) \) and plotting:

\[
I = g_l(V - V_l) + g_k w_\infty(V)(V - V_k) + g_{Ca} m_\infty(V)(V - V_{Ca})
\]

2. Draw the phase plane for \( I = 80 - \) a good window is \(-70 < V < 60\) and \(0 < w < 1\). Draw some trajectories. Set the total amount of time to 150 msec. (Nullclines should be plotted. If you use XPP, then click on Nullcline New. If you use MATLAB, you should plot \( w = w_\infty(V) \) as the \( w \)–nullcline and

\[
w = \frac{I - g_l(V - V_l)g_{Ca} m_\infty(V)(V - V_{Ca})}{g_k(V - V_k)}
\]

as the \( V \)–nullcline.)

3. Set \( I = 120 \) and draw the nullclines. Find the periodic orbit and estimate the period.

4. Set \( I = 90 \) and show that the cell is bistable. Compute the stable oscillation and the unstable oscillation. (To compute the unstable oscillation for a planar system, change \( dt \) to a negative step to integrate backwards in time.) Don’t forget the nullclines!

5. Try to compute the bifurcation diagram with \( I \) as a parameter between 60 and 250.

Now, set \( vc = 12, vd = 17.4, \phi = 0.066666, gca = 4.4 \).

1. Set \( I = 25 \) and window \(-.25 < w < 1\). Sketch the nullclines and sample trajectories. The rest point about \((-45, 0)\) Starting with \( w(0) = 0 \), what is the voltage threshold to get a spike? As you approach the threshold, the time to spike can get quite long!

2. Find the minimum current that generates periodic spiking. You may have to integrate for a long time.
3. Try to compute the bifurcation diagram.

The following link may be of use! It gives parameters etc for AUTO and if you use it, XPP