MATH 3375 - Assignment 1

1. Derive the Nernst potential from the equilibrium point for the Nernst-Planck equation:

\[ 0 = uzRT \frac{d[C]}{dx} + uz^2F[C] \frac{dV}{dx} \]

given that \([C](0) = C_{in}, V(0) = V_{in}\) and \([C](l) = C_{out}, V(l) = V_{out}\) where \(l\) is the thickness of the membrane. By definition, \(V_{eq} = V_{in} - V_{out}\). Using the formula, calculate \(V_{Na}\) at \(T = 10, 20, 37\) degrees centigrade, given that \([Na]_{in} = 10mM\) and \([Na]_{out} = 145mM\). Use the following values, \(F = 96485.3C/mol, R = 8.3145j/(mol\cdot Kelvin)\) and Kelvin = centigrade + 273.16.

2. The constant field equation (CFE) gives the current voltage relationship when the concentrations of ions are taken into consideration:

\[ I = PzF\xi \left( \frac{[C]_{out}e^{-\xi} - [C]_{in}}{e^{-\xi} - 1} \right) \]

where \(\xi = zVF/(RT)\). Plot the \(I-V\) curve using this equation for sodium at 20°C using concentrations from the previous exercise. Note that the current is not a straight line, but is rectified. Plot the same curve for potassium using \([K]_{in} = 140mM\) and \([K]_{out} = 5mM\). Finally, given the inside and outside concentrations of sodium, potassium, and chloride, as well as their permeabilities, \(P_{Na}, P_{K}, P_{Cl}\), find the Goldman-Hodgkin-Katz (GHK) equilibrium using the CFE by solving

\[ 0 = I_{Na}(V) + I_{K}(V) + I_{Cl}(V) \]

for \(V\). Careful that you note that sodium and potassium have valence +1 but chloride has valence -1.

3. Solve the following driven passive membrane:

\[ c \frac{dV}{dt} = gL(V_{eq} - V) + I\cos(\omega t) \]

and find the steady state response. It will be of the form:

\[ V_{ss}(t) = V_{eq} + A \cos(\omega t) + B \sin(\omega t) \]

and compute the power, \(\sqrt{A^2 + B^2}\).

4. Compute by simulation the firing rate vs current (FI) curve for the leaky integrate-and-fire model (LIF):

\[ c \frac{dV}{dt} = -gL(V - V_m) + I \]

with \(V_m = -65mV, V_{thr} = -50mV, V_{reset} = -70mV, c = 1\) for \(gL = 0.01\) and \(gL = 0.05\). Vary the current from 0 to 2. For the case \(gL = 0.01\),
compute the FI curve when there is a refractory period of 3 msec. That is, once the neuron fires, it cannot fire again for 3 msec. Note that you should plot the firing rate in units of Hertz. Since the units here are milliseconds, make sure you multiply by 1000. The easiest way to do this is to simulate it for, say, 2000 msec and count the number of spikes.

5. Simulate the above LIF with all parameters as above and no refractory period with a double current pulse. That is inject a 3 msec pulse of current with amplitude 2 at $t_1 = 20\text{msec}$ and then again at a later time, $t_2$. A single pulse is not enough to cause the neuron to fire. Use $g_l = 0.05$ and find (by simulation), the maximum value that $t_2$ can be to elicit a spike. E.g. if $t_2 = 21$ then a spike will be emitted and $t_2 = 100$ it won’t be. Find the largest value of $t_2$ that will yield a spike. Do the same for $g_l = 0.02$. In the “high conductance” state, the neuron is a “coincidence detector” and requires the inputs to be in close proximity, but in the “low conductance” state, it is an “integrator.”

6. Simulate the quadratic integrate and fire mode (QIF):

$$cdV/dt = g_L(V - V_m)(V - V_{th})/(V_{th} - V_m) + I$$

with $V(t) \to V_{reset}$ when $V(t) = V_{spike}$. Use $c = 1, V_m = -65, V_{th} = -55, g_L = 0.05, V_{spike} = 20, V_{reset} = -70$. Try $I = 0.2$ and $I = 1$. Compute the FI curve for $I \in [0, 2]$.

7. Analytically compute the FI curve for the dimensionless QIF model with infinite spike and reset:

$$v' = v^2 + I$$

where $v(t) \to -\infty$ when $v(t) = +\infty$. To do this, compute the time it takes to get from $-\infty$ to $\infty$:

$$T = \int_{-\infty}^{\infty} \frac{1}{v^2 + I} dv.$$ 

If you need some help in the programming, you can ask me. I don’t know Matlab, so I do it all using my software.