Answers to HW 2 (49 Points, I think)

1. **8 total points** (a) **3 pts** Atmospheric drag force is proportional to $Sv^2$ and gravitational force is $mg$. Assuming density is constant, then $S \propto V^{2/3} \propto m^{2/3}$. Balance of forces implies 

$$Sv^2 \propto m^{2/3}v^2 \propto mg \quad \text{(8 points total, I think)}$$

so that $v^2 \propto m^{1/3}$, or $v \propto m^{1/6}$. (b) **2 pts** Show that kinetic energy per unit area scales as $m^{2/3}$. KE is proportional to $mv^2 \propto m^{4/3}$ from the above. Surface area is proportional to $m^{2/3}$ as above, so KE per area scales as $m^{4/3}/m^{2/3} = m^{2/3}$ as required. (c) **1 point** KE has to go somewhere when the animal falls so big animals are hurt more so than small animals as the KE grows pretty fast with mass (d) **2 points** Potential energy is $mgh$ and KE is $mv^2/2$ so we get $h \propto mv^2/m = v^2 \propto m^{1/3}$ as required.

2. Turkey problem **10 points total** (a) **4 points** Graph via excel gives an estimate for the time to roast a turkey as $t \propto m^{0.578}$ which is not very close to the rule of thumb of $x$ minutes per pound. (b) **6 points** Let $u, u_O, m, \rho, t, \kappa$ be temperature of the turkey, oven temperature, mass, density, time to cook, heat capacity ($L^2/T$ dimensions). Let $M, L, T, K$ be dimensions of mass, length, time, and temperature. Then 

$$u^a u_O^b m^c \rho^d T^e \kappa^f = K^{a+b} M^{c+d} L^{-3d+2f} T^{-f}$$

This gives $a = -b, c = -d, f = 3d/2, e = 3d/2$, so we could set $d = 0$ and $b = 1$ to get 

$$\pi_1 = u/u_O$$

as one dimensionless group. The second one is in terms of $d$, so setting $d = 2$, we get $c = -2, f = 3, e = 3$ and hence 

$$\pi_2 = \frac{\rho^2 \kappa^3 t^3}{m^2}$$

We must have $F(\pi_1, \pi_2) = 0$ which can be solved as $\pi_2 = g(\pi_1)$ from which we see that:

$$t^3 = \frac{m^2 g(\pi_1)}{\kappa^3 \rho^2}$$

If $\kappa, \rho, \pi_1$ are all independent of $m$, we obtain $t \propto m^{2/3}$ as desired! This is much closer to 0.578 than 1 is. So it is better.

3. **5 points** Let $v, g, \lambda, y$, be the velocity of the wave, gravitational acceleration, wavelength, and depth of the wave. We have 

$$v^a \lambda^b \gamma^d = (L/T)^a (L/T^2)^b L^c L^d = L^{a+b+c+d} T^{-a-2b}$$

so, we get $a = -2b, c = -d + b$. Set $d = 0, b = -1$ (could set $b = 1$ to get the same result, but this is more compact) to get $(a, b, c, d) = (2, -1, -1, 0)$ yielding 

$$\pi_1 = \frac{v^2}{\lambda g}$$

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and set $b = 0, d = 1$ to get $(a, b, c, d) = (0, 0, -1, 1)$ yielding

$$\pi_2 = \frac{y}{\lambda}$$

Thus, $F(\pi_1, \pi_2) = 0$ or, $\pi_1 = h(\pi_2)$ which implies

$$v = \sqrt{\lambda g f(y/\lambda)}$$

where $f = sqrt(h)$ as required.

4. Problems from Meerschaert

(a) Problem 1. **14 points total** (a) **5 points** Let $r$ be the rebate in units of 100$. Let $S$ be the sales without a rebate. Let $P$ be the profit that the dealer gets. The profit that the dealer gets from each car is $1500 - 100r$ (so rebate of $r = 1$ costs the dealer $100$). Each such rebate increases sales by 15%, so that the total sales should be $S(1 + 0.15r)$ with the rebate. In anticipation of the second part of the question, I will call the 15% boost (0.15 fractional boost), $b$, so that the total profit is sales times profit per car:

$$P = (1500 - 100r)S(1 + br)$$

with $b = 0.15$ for us. (Note - you should give partial credit for setting this up correctly. There are other ways you could do it, such as the rebate in dollars instead of 100 dollar amounts) Differentiation of this with respect to $r$ gives:

$$r = \frac{15b - 1}{2b}$$

which gives $r = 4.1666$ and an answer of rebate of $416.66$ rebate. Substitute this into the profit to get

$$P = 25S(15b + 1)^2/b$$

for the profit of 1760.41S. (b) **4 points** We can use the above to compute the sensitivity.

$$\frac{dr}{db} = 1/(2b^2)$$

So we get the sensitivity, $(b/r)(dr/db)$

$$S_r = 1/(15b - 1)$$

For $r = 0.15$, we get 0.8, so it is amplified by quite a bit. For the profit, we have

$$\frac{dP}{db} = S(5625b^2 - 25)/b^2$$
And from this

$$S_P = \frac{(15b - 1)}{(15b + 1)}$$

which is about 0.38 for \( b = 0.15 \) (c) \textbf{3 points} If the rebates generate just 10\% then the rebate is only $250 and the sensitivity is a whopping 2. The profit is $1562.50 S, considerably less than in (a). If the response is in between, then the rebate will also be in between and similarly with the profit. (d) \textbf{2 points} Once the rebate gets above \((15b - 1)/b\), the profit will be less. For \( b = 0.15 \), this is $833 rebate.

(b) Problem 7 in the book. \textbf{12 points total} (a) \textbf{6 points since algebra is horrible} Since we will have to look at sensitivity to growth rate and price drop, we will keep these as parameters. From the earlier pig results, we have a pig that is 200 lb and now grows at \( g = 5 \) lbs/day. We’ve had him for 90 days and invested $100 in him already. He costs $.45 a day to maintain. So with same notation and \( F \) the profit per day

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\begin{align*}
C &= 100 + 0.45t \\
p &= 0.65 - at \quad (a = 0.01) \\
w &= 200 + gt \quad (g = 5) \\
R &= pw \\
P &= R - C \\
F &= P/(90 + t)
\end{align*}
\]

(Note that it is really important that they divide by \((90 + t)\) and not \( t \) since it is ill defined then!) Using Maple, to maximize \( F \), I find, \( t = 4.55 \) is the best time to sell and the profit rate is about 34 cents/day. (b) \textbf{3 points}. To get the sensitivity with respect to \( a \) I will compute \((a/t)dt/da\) and \((a/F)dF/da\). Using Maple, I get \( S(t, a) = -5.15, S(F, a) = -.3109 \) As expected if \( a \) increases I should sell sooner and the profit will decrease. (c) \textbf{3 points}. \( S(t, g) = 5.82, S(F, g) = 0.42 \) They dont have to show work on these.