Answers to HW 9 34 points

1. **4 points** Find all the Nash equilibria for the following payoff matrix

\[
\begin{pmatrix}
(5, 5) & (5, 10) & (8, 6) \\
(6, 8) & (4, 4) & (1, 3) \\
(3, 1) & (3, 1) & (7, 7)
\end{pmatrix}
\]

Answers (6, 8), (5, 10) 4 pts

2. **Asymmetric rock paper scissors. 15 pts** Using the replicator equations, letting \( x, y, z \) be the fraction playing rock, paper, scissors, we get the fitnesses: (note \( z = 1 - x - y \))

\[
\begin{align*}
f_x &= -ay + bz \\
f_y &= bx - az \\
f_z &= by - ax
\end{align*}
\]

The mean fitness is \( \phi = xf_x + yf_y + zf - z \) and the equations (5 pts for the equations) are:

\[
\begin{align*}
x' &= x(f_x - \phi) \\
y' &= y(f_y - \phi)
\end{align*}
\]

Equilibria that are non-negative are \((0, 0), (1, 0), (0, 1), (1/3, 1/3)\) (4 points) There are others but they are not relevant as they will not all lie between 0 and 1. Don’t take points off for including them. You can use a computer algebra system to get the stability. For the pure equilibria \((0, 0), (1, 0), (0, 1)\) the Jacobian is triangular or diagonal and the eigenvalues are \(-a, b\) and since \(a, b\) are positive, these are always saddles. 3 points)

3. **Mixed strategy for Hawk dove. 15 points** Play Hawk with \( p \) and Dove with \( q = (1 - p) \) Here is the table (it is symmetric, so don’t need the pairs, but is you want to do pairs then the diagonal entries are of the form \((x, x)\) where \(x\) is what is in the table and the off diagonal entries are \((x, y), (y, x)\) ) 9 points, one for each correct entry

\[
\begin{array}{ccc}
H & D & M \\
\hline
H & (G - C)/2 & G \\
D & 0 & G^2/2 \\
M & p(G - C)/2 & (1 - p)G^2/2 \\
pG + (1 - p)G^2/2 & p^2(G - C)/2 + p(1 - p)G + (1 - p)^2G^2/2 \\
\end{array}
\]

To find a mixed strategy that is a Nash equilibrium, we want to choose \( p \) so that the \((M, M)\) is at least as good as \((D, M)\) and \((H, M)\); that is choose
$p$ so that

$$p^2(G - C)/2 + p(1 - p)G + (1 - p)^2G/2 \geq (1 - p)G/2$$
$$p^2(G - C)/2 + p(1 - p)G + (1 - p)^2G/2 \geq p(G - C)/2 + (1 - p)G$$

The first inequality requires $p \leq G/C$ and the second $p \geq G/C$ so this means $p = G/C$ is the mixed strategy. (4 points) For the case $G = 1, C = 2, p = 0.5$. 2 points