Sample Exam 1 - Math 1270

1. Solve the following initial value problems

   (a) $y' - 3y/t = t^2$ with $y(1) = 2$

   (b) $t^2y'' - 4ty' + 6y = 0$ with $y(1) = 0$ and $y'(1) = 1$ (An euler equation,p166)

   (c) $y' = 1 + y^2$ with $y(0) = \alpha$. For each α compute the interval of existence $T_1 < t < T_2$ for the solution.

   (d) $y' + 2y/t = y^3$ with $y(1) = 1$. What is the interval of existence for the solution to this problem? (Note this is a Bernoulli equation, p77)

   (e) $y'' + 6y' + 8y = te^{-t}$ with $y(0) = 0, y'(0) = 1$.

   (f) $y'' + 9y = \csc(3t)$

   (g) $dy/dx = (x^2 + 3y^2)/(2xy)$ (see page 49-50)

2. Match the direction fields shown in the figure with the correct differential equation, $y' = f(t, y)$. Here are the choices for $f(t, y)$: (i) $t - 1$, (ii) $1 - y^2$, (iii) $y^2 - t^2$, (iv) $1 - t$, (v) $1 - y$, (vi) $t^2 - y^2$, (vii) $1 + y$, and (viii) $y^2 - 1$. Explain your reasoning. Also, sketch a representative trajectory forward and backward in time with initial condition $y(0) = 3/2$. You do not need a computer to solve this.

3. Let $f(y) = (y-2)^2(y-3)(y-5)(y-6)$. Find all the equilibrium points to $y' = f(y)$ and determine their stability. In the (t, y) plane, sketch solutions with initial conditions $y(0) = 1.9, 2.1, 3, 5.5$.

4. Solve the differential equation:

$$(yx^2 - y)dy + (xy^2 - 2x^3 + x)dx = 0.$$

5. If you attempt to tap your fingers in an alternating rhythm, as the frequency at which you tap increases, there is a sudden switch from alternate to synchronous tapping. Kelso and Haken proposed the following model for the phase-difference, ϕ, between your two fingers:

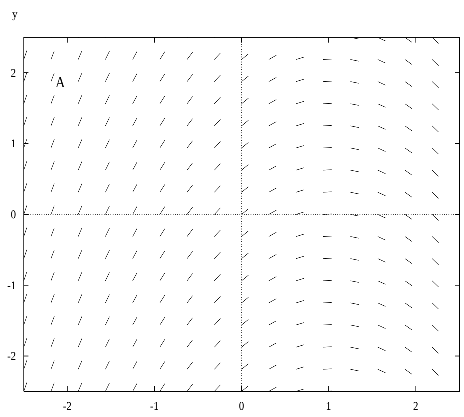
$$\frac{d\phi}{dt} = -\sin\phi - \lambda \sin 2\phi.$$

   They suggest that as the frequency goes up, the parameter λ decreases. Find all the equilibria. Sketch the phase line when $\lambda = 0$, $\lambda = 1/2$, and $\lambda = 1$ indicating all stable and unstable equilibria in the interval $[0, 2\pi]$.

6. Consider the differential equation

$$y'' + p(t)y' + q(t)y$$

   with continuous coefficients on an interval (a, b). Let $y_1(t), y_2(t)$ be two nonzero solutions such that they both have a local maximum at $t = t_0 \in (a, b)$. Show that they do not form a fundamental set of solutions and that one is linearly related to the other.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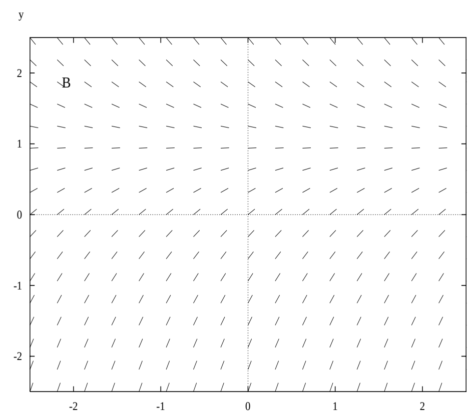
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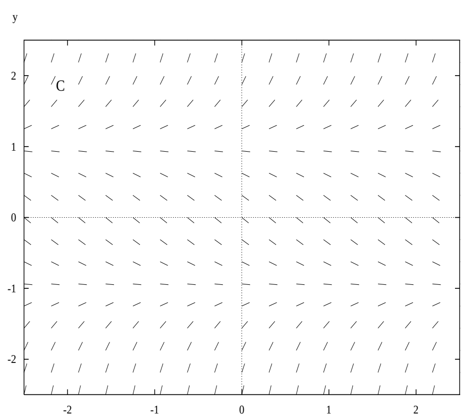
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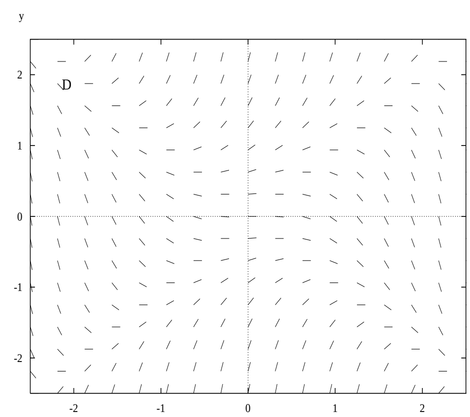
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