

Homework Due Monday April 12

1. Do Problems 1,2 on page 5 of the handout on simple mechanical systems (the problems with the graphics of potential functions)
2. For each of the following determine if the system is a gradient or hamiltonian system and find the appropriate function:

(a) $x' = y + 2xy, y' = x + x^2 - y^2$

(b) $x' = -3x - 4y^3, y' = 2x + 3y$

(c) $x' = \cos(x) + y, y' = -\sin(y) + x$

(d) $x' = x^2 - 2xy, y' = y^2 - x^2$

(e) $x' = x^2 - y^2, y' = x^2 - 2xy.$

3. Prove that the linearization of a fixed point for a hamiltonian system has eigenvalues of the form $\pm a$ or $\pm ia$ where a is real.
4. Find a condition on the matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

such that the system $X' = AX$ is (i) hamiltonian; (ii) gradient.

5. Miscellaneous old stuff that I forgot to give you. Use Fulmer's method to compute the matrix exponential for the following matrices:

$$A_1 = \begin{pmatrix} -2 & 5 \\ -1 & 2 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -3 & 1 \\ -1 & 1 \end{pmatrix}$$