Homework – Due mar 26

1. For each of these problems, find all the fixed points and determine their stability

   (a)

   $$x' = y - x^2 \qquad y' = x - y$$

   (b)

   $$x' = x - x^3 - y \qquad y' = x$$

   (c)

   $$x' = x + y \qquad y' = xy + 1$$

   (d)

   $$x' = x(1 - x - y) \qquad y' = y(-1 + x/y)$$

   Note that this is a predator prey model

2. The following is a model for the spread of an infection; S is the susceptible population; I is the infected and β is the infectivity. The equations are:

   $$S' = -\beta SI + 1 - S \qquad I' = \beta SI - I$$

   Find all the fixed points and their stability. For what values of the parameter β does there exist a stable *endemic*, that is a stable nonzero level of infecteds, I.

3. Consider the quadratic differential equation:

   $$x' = y \qquad y' = a - y - x^2 + xy$$

   Find all the fixed points, their stability, and find all values of a where the fixed points are nonhyperbolic.

4. The following is a genetic control system. x is the proportion of protein and y the proportion of messenger RNA:

   $$x' = y - ax \qquad y' = \frac{x^2}{1 + x^2} - y$$

   The parameter a is the decay rate and is positive. For what values of a are there three fixed points?