How do we plot the integral curves of the equation:

$$M(x,y)dx + N(x,y)dy = 0?$$

We write this as a pair of differential equations:

$$\begin{aligned} \frac{dx}{dt} &= N(x,y) \\ \frac{dy}{dt} &= -M(x,y). \end{aligned}$$

To see why this is true, divide the two equations:

$$\frac{dx/dt}{dy/dt} = -\frac{N(x,y)}{M(x,y)}$$

By the chain rule, this becomes

$$\frac{dx}{dy} = -\frac{N(x,y)}{M(x,y)}$$

or

$$M(x,y)dx = -N(x,y)dy$$

implying

$$M(x,y)dx + N(x,y)dy.$$

Example. Consider

$$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}$$

which we rewrite as:

$$-(3x^2 + 4x + 2)dx + 2(y-1)dy = 0.$$

Thus, $M = -(3x^2 + 4x + 2)$ and $N = 2(y-1)$. Now here is how to do this with my free software XPPAUT.

1. use a text editor to create a plain text file which has the following in it. (Save it as plain text, not DOC, not RTF, just text!)

   ```
   M(x,y)=-(3*x^2+4*x+2)
   N(x,y)=2*(y-1)
   # this is example 2, page 44
   dx/dt=N(x,y)
   dy/dt=-M(x,y)
   # this sets up plotting
   @ xp=x,yp=y,xlo=-3,xhi=3,ylo=-3,yhi=3
   done
   ```

2. Run XPP (in windows, drop the file into the desktop shortcut), in Linux, you can run it via the command line *XPP ex2-44.ode*.

3. To solve specific initial conditions, click on **Init Conds New** and type them in, say *x=0,y=-1.* Then save this curve by calling **Graphics Freeze Freeze** and choose color 1 (red). Note this only solves forward. Click **Init Conds Backward** to solve backward in time and freeze this curve.

4. To get the big picture, click on **Dir.Fld/Flo Flow** and choose 5 for the grid size. You will get something like fig 2.2.2 in the book.

5. To get direction fields, click **Dir.Fld/Flow Scaled DirFld** and choose 10 for the grid.

6. To get hard copy of it you can click Alt PrtSc in Linux or Windows. In Windows, it copies it to the clipboard and in Linux, creates a PNG. In either case, it is printable.

7. To change M, N, just click **File Edit Functions.**

8. To change the window size, **Window Window** and fill in the sizes.

We can also plot the direction fields for problems of the form:

$$\frac{dy}{dt} = f(t, y)$$

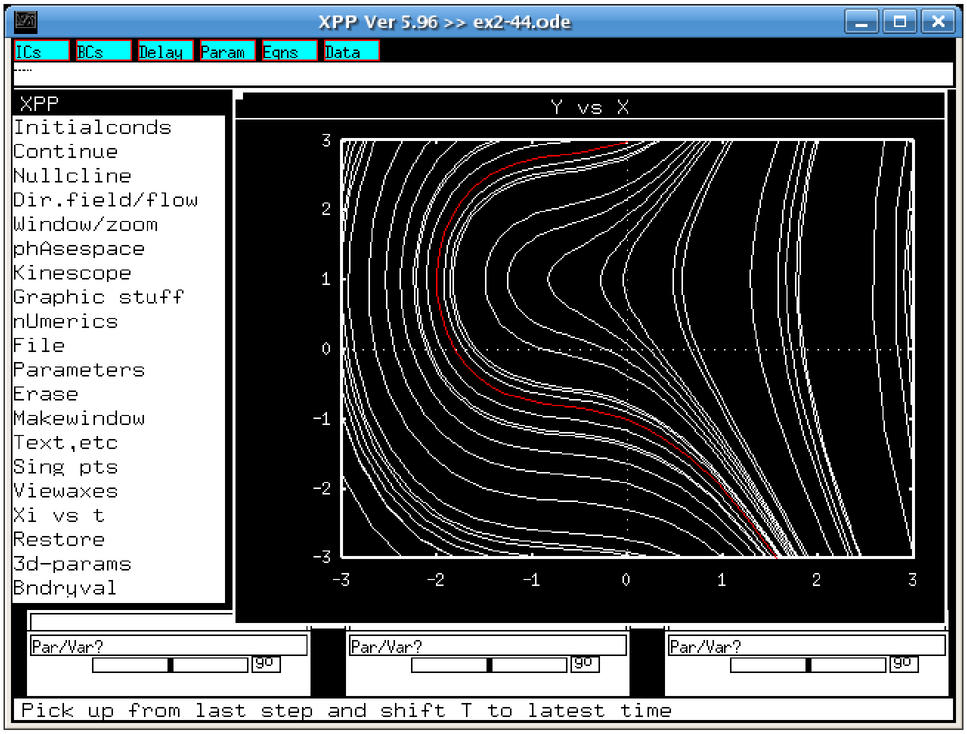
by making them depend on x where x is the same as time. That is

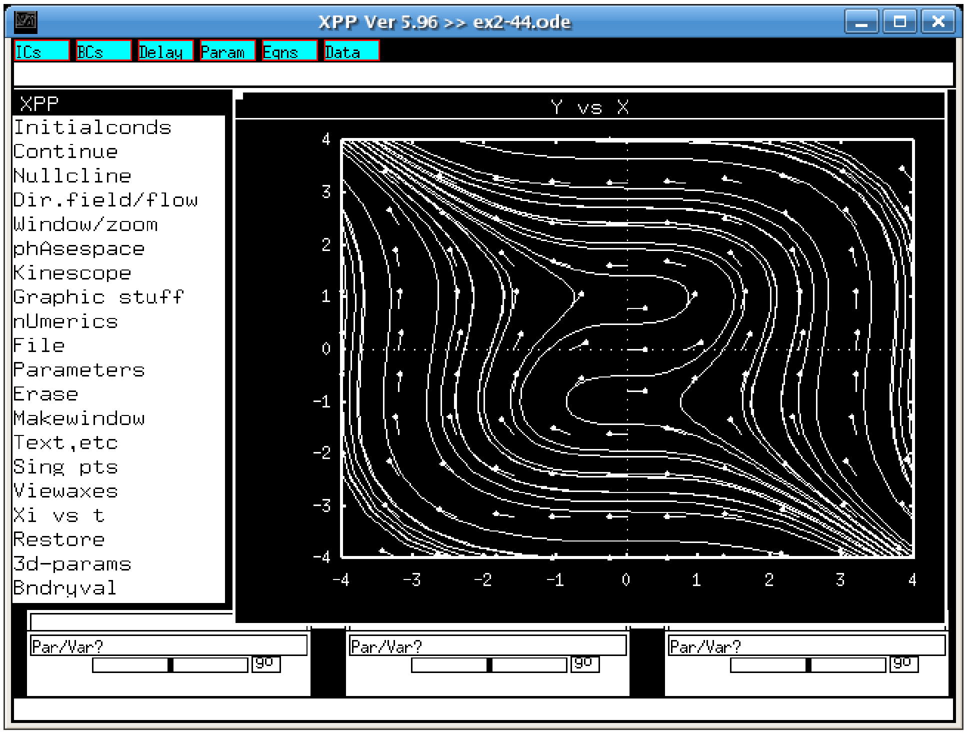
$$\begin{aligned} \frac{dx}{dt} &= 1 \\ \frac{dy}{dt} &= f(x, y) \end{aligned}$$

Now, this looks like the above and we can get direction fields in the same way. Here is the code:

```
f(t,y)=sin(t)-y
x'=1
y'=f(x,y)
@ xp=x,yp=y,xlo=-2,xhi=8,yhi=2,ylo=-2
done
```

Just run this and draw the direction fields by clicking **Dir. Fld/Flow Scaled Dir Fld** and choose **16** for the grid. Save and pront as above. To change the function $f(t, y)$, just click on **File Edit Functions** and type in the new one.


XPP Ver 5.96 >> ex2-44.ode
ICs
BCs
Delay
Param
Eqns
Data
XPP
Initialconds
Continue
Nullcline
Dir.field/flow
Window/zoom
phAsespace
Kinescope
Graphic stuff
nUmerics
File
Parameters
Erase
Makewindow
Text,etc
Sing pts
Viewaxes
Xi vs t
Restore
3d-params
Bndryval
Y vs X
Par/Var?
go
Pick up from last step and shift T to latest time


XPP Ver 5.96 >> ex2-44.ode
ICs
BCs
Delay
Param
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Data
XPP
Initialconds
Continue
Nullcline
Dir.field/flow
Window/zoom
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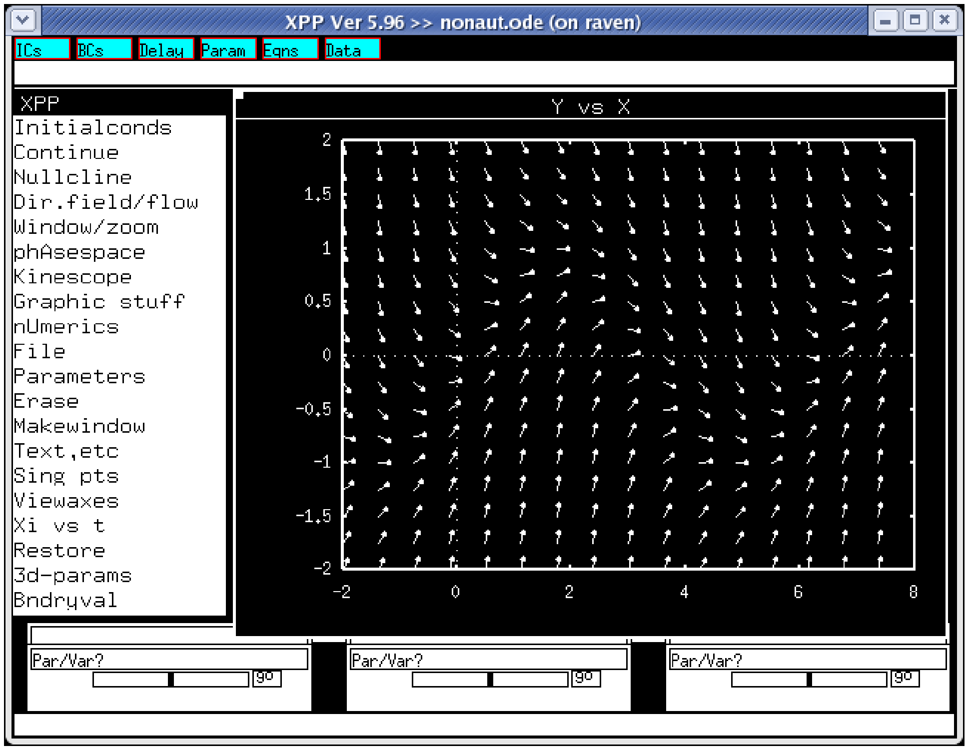
Kinescope
Graphic stuff
nUmerics
File
Parameters
Erase
Makewindow
Text,etc
Sing pts
Viewaxes
Xi vs t
Restore
3d-params
Bndryval
Y vs X
Par/Var?
go
Par/Var?
go
Par/Var?
go


XPP Ver 5.96 >> nonaut.ode (on raven)
ICs
BCs
Delay
Param
Eqns
Data
XPP
Initialconds
Continue
Nullcline
Dir.field/flow
Window/zoom
phAsespace
Kinescope
Graphic stuff
nUmerics
File
Parameters
Erase
Makewindow
Text,etc
Sing pts
Viewaxes
Xi vs t
Restore
3d-params
Bndryval
Y vs X
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
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2
4
6
8
Par/Var?
go
Par/Var?
go
Par/Var?
go