Sample Final

1. Find an equation for the plane that passes through the point $(2, 3, 7)$ and contains the line parametrized by $r(t) =< 1 + 5t, 0, -2t >$.

2. Let F be the two-dimensional vector field given by

$$F(x, y) = \langle ye^{xy} - 1, xe^{xy} + 2y \rangle .$$

   (a) Find a potential function $f(x, y)$ for F.

   (b) Find the value of the line integral

$$\int_C F \cdot dr$$

   where C is the line segment from (0, 3) to (5, 0).

3. Let f be the function of three variables given by $f(x, y, z) = (x+3y-z)^5$.
(a) Find the linearization $L(x, y, z)$ of $f(x, y, z)$ at the point (2, 1, 4).

   (b) Give the equation of the tangent plane to the level surface $f(x, y, z) = 1$ at the point (2, 1, 4).

   (c) Use the linearization you found in part (a) to give an approximate value for $f(2.01, 0.98, 4)$.

4. (a) Show that there is no vector field G such that curl$G =< 2x, 3xy, -xz^2 >$

   (b) Suppose that $\nabla^2 f = 0$. Show that the line integral $\int_C f_y dx - f_x dy$ is independent of path in any simple region, D.

5. Let C be the ellipse with equation $x^2 + 4(y - 6)^2 = 25$. Find the point(s) of C that are closest to the origin and the point(s) of C that are farthest from the origin. For each of these points give also the distance from the point to the origin. (Hint: $17 < 2\sqrt{73}$.)

6. The equation $z = r^2$ in cylindrical coordinates describes a surface in R^3 . Let S be the portion of this surface that lies between the two planes $z = 1$ and $z = 4$.

   (a) Give a parametrization for the surface S. (Remember to include the domain of the parametrization.)

   (b) Find an iterated integral giving the total surface area of S. You should simplify the integrand as much as possible, but you need not evaluate the integral.

7. Find the value of the line integral

$$\int_C -5x^2 dx + 7xy dy$$

   where C is the closed curve consisting of the edges of the triangle with ver- tices (0, 0), (3, 1) and (0, 3), oriented counterclockwise.

8. Find the total flux $\int\int_S F \cdot N d\sigma$ of the vector field

$$F = \langle x^2, yz^2, -2xz \rangle$$

across the surface S given by $x^2+y^2+z^2 = 2$, with the outward orientation.