A New Kind of Science; *Stephen Wolfram*, Wolfram Media, Inc., 2002. Almost twenty years ago, I heard Stephen Wolfram speak at a Gordon Conference about cellular automata (CA) and how they can be used to model seashell patterns. He had recently made a splash with a number of important papers applying CA to a variety of physical and biological systems. His early work on CA focused on a classification of the behavior of simple one-dimensional systems and he published some interesting papers suggesting that CA fall into four different classes. This was original work but from a mathematical point of view, hardly rigorous. What he was essentially claiming was that even if one adds more layers of complexity to the simple rules, one does not gain anything beyond these four simples types of behavior (which I will describe below). After a two decade hiatus from science (during which he founded Wolfram Science, the makers of *Mathematica*), Wolfram has self-published his *magnum opus* **A New Kind of Science** which continues along those lines of thinking. The book itself is beautiful to look at with almost a thousand pictures, 850 pages of text and over 300 pages of detailed notes. Indeed, one might suggest that the main text is for the general public while the notes (which exceed the text in total words) are aimed at a more technical audience. The book has an associated web site complete with a glowing blurb from the publisher, quotes from the media and an interview with himself to answer questions about the book. The copyright notice in this book is somewhat unusual – essentially saying that if you use any of the ideas in the book, you had better cite him or his lawyers will see you. (It should be noted that there are *no* citations in this book.)

Wolfram makes many points in his book, but I will focus on three of them: (i) complex behavior can be generated by simple rules; (ii) modern mathematical methods are ill-equipped to deal with complex systems such as seen in biology and social sciences; and (iii) all systems can be viewed as computations and furthermore, even the most complex ones are equivalent to certain simple rules. This latter notion is the core idea in the book.

The book begins with an outline of what will come later. Here Wolfram describes his main thesis: The Principle of Computational Equivalence. He bases almost everything in the book on countless computer experiments using cellular automata (CA). One should recall that a CA consists of a lattice of sites each of which has an associated state (often just 0 or 1). Each site is updated according to a rule which generally depends on some small neighborhood of nearby sites. For years CA were viewed mainly as a parlor game for producing pretty pictures. One of the goals of the present book is to claim that rather than just a simple game, these systems are how we should all be doing science. As an example consider a one-dimensional lattice with two states which are updated according to their current values and the two neighboring sites. There will be 256 possible rules so it is not at all difficult to exhaustively explore each possible rule. Wolfram provides a clever numerical code for specifying these rules. He is partial to two of these rules, 30 and 110. The binary representation of 110 is 01101110. The set of three digits consisting of your state and your two neighbors encodes a binary number from 0 to 7. Thus if this number is 0,4, or 7, then your new state will be zero otherwise, it will be 1. That is, {000, 100, 111} are mapped to 0 and all others are mapped to 1.

Chapter three introduces many different variants on one-dimensional cellular automata and then goes on to introduce many other simple rule based systems. Again and again, Wolfram refers to the striking result *he has discovered*, namely, that simple rules can generate complicated behavior. This is hardly a novel observation as countless students who have coded the discrete logistic map into their calculators can attest. Wolfram makes an interesting point about the pictures that the CA produce. He asks (in the Notes, p.860) how one might attempt to infer the rules from the pictorial output and claims that standard analysis methods would not work. I am not sure he is right here; for the one-dimensional CA rules with a few nearest neighbors, one could look at correlations of a cell at position i at time t with the states of those in some neighborhood of i at $t-1$ just as it has been done for years with experimental data. Many new types of discrete machines are introduced and some of them will later be used in the proof of the universality of rule 110 (see below). I should point out that Wolfram introduces hundreds of simple rule based systems throughout the book and provides the source code for these in *Mathematica* format. Regrettably, unless you are an expert in Mathematica, fragments of code like `NestList[f[RotateRight[#],#]&,init,t]` will not be particularly helpful.

In chapter four, he introduces the reader to number systems and shows the kinds of CA that emerge when simple arithmetic rules are applied to base 2 representations of numbers on a lattice. He devises some clever rules which generate the binary square roots of numbers as well as a CA version of the Sieve of Eratosthenes. He implements some classic partial differential equations such as the diffusion equation and the sine-Gordon equation using CA. These are all amusing games, but do little to convince this reader of the usefulness of CA as computational tools. He closes this chapter with an argument for the use of discrete dynamics as a modeling tool and begins his polemic against classical continuous models in science. Indeed, this is one of the major themes to emerge from the book – classical mathematics is the wrong way to approach the study of complex behavior in Nature.

He pays lip service to higher dimensional CA models (e.g two- and three-dimensional lattices) in Chapter five. His main point here is that although the dimensions are different, the behavior of his simple rules in higher dimensions is not much different from the one-dimensional case. He illustrates this point nicely by displaying the evolution of one-dimensional cross-sections of two-dimensional models. He introduces fractals and the kinds of affine maps made famous by Barnsley and his colleagues [1].

Wolfram's major contribution to the theory of CA is presented in Chapter six where he starts his automata with random initial data. Here he outlines his "classification" of the possible behavior in CA models. The classification is based on exhaustively simulating all possible rules that involve a set number of neighbors and a fixed number of states. His finding (and this is very old news; it was published in the early 1980's) is that there are 4 classes: (I) everything is all black or white (e.g. rule 58) ; (II) periodic phenomena in "space" or "time" (rules 200,208); (III) random and self-similar structures (44,122) ; (IV) a mixture of order and randomness (110,20). (Note, the coding scheme is as described above for rule 110.) One of the “hallmarks” of class III automata is that local changes are evenutally propagated to the rest of the medium. Classes I and II never show such propagation and class IV may or may not propagate disturbances. The classification is done by eye and as far as I can tell, there is no way to assign a class to a particular rule in any quantitative manner. This kind of vagueness throughout the book becomes increasingly annoying as one works through the various chapters. I suppose that this is part of his point; if there were a statistical way to assign the classification, then it would presumably fall into the realm of traditional scientific and mathematical methods. However, that puts one squarely in the pre-Newtonian age of purely descriptive science. Indeed, this descriptive ill-defined methodology occurs throughout the book; one of the main concepts, “complexity” is never defined in a satisfactory manner.

Chapters six through eight attempt to apply the descriptive behavior of the previously discussed simple rules to models of natural phenomena. This is what Wolfram regards as the “new science” in the title of the book: replace the continuum theories developed over the last three-hundred years with simple programs such as automata. His first scientific method is to compare pictures of natural phenomena with the pictures of his CA models. Anyone who has been in the field of pattern formation recognizes this approach and its relative merits and failures. Take for example the problem of periodic patterns seen in many biological systems [5]. The mechanisms underlying spontaneous symmetry breaking are understood and almost universal: lateral inhibition. However, the job of the modeler and the scientist is subtler; she must devise ways to determine the implementation of this general mechanism given the constraints of the system. For example, it is unlikely that the stripe formation observed in occular dominance columns seen in the visual cortex of many mammals is due to mechanical effects like those used to model limb formation. These aspects of science are neglected in the present volume as if details are irrelevant.

The nature of randomness and its causes (intrinsic versus environmental) are discussed and the relationship of CA models to chaos are covered in this chapter. Wolfram points out that it may be that the randomness seen in nature is due more to intrinsic (that is deterministic) randomness rather than external randomness. He points out that the random number generator used in Mathematica is one of his CA models.

Chapter seven suggests implications of his preceding ideas for everyday systems. Like much of the book, each “application” is sketchy and shallow with few or no testable hypotheses. Furthermore, most of the examples are old standards (snow flakes, animal coats) and so not seem to make his point very convincingly. Here is a quote from the book which will probably offend most of my biological colleagues and about summarizes Wolfram’s philosophy of modeling (p. 365):

> Typically it is not a good sign if the model ends up being almost as complicated as the phenomenon it purports to describe. And it is an even worse sign if when new observations are made the model constantly needs to be patched in order to account for them.

While I agree with the first part of this quote, the second is the antithesis of how science is done. Hypotheses (particularly in biology) are continually refined as data gets better and better. Wolfram's approach to this "New Science" is to create the simplest model which "looks like" the phenomena and stop there. Again, he wants us to move back into the purely descriptive age of science. A page is spent claiming that (i) he has constructed but never published at least a dozen significant models and that (ii) in spite of their simplicity experts have come to believe they are right. However, almost no references to the models or experts are given. He uses CA to model crystal growth, the fracturing of materials, fluid flow, plant growth, phyllotaxis, the shapes of sea shells, and pigmentation patterns of shells. This last example is probably the best known among theoretical biologists as he presented his work on sea shell pigmentation at a Gordon Conference in the early 1980's. Indeed, his class three cellular automata were the first examples of a dynamical system which could produce the kinds of nested triangle patterns seen on molluscs like *Oliva porphyria*. Automata models of seashell patterns actually have a history going back to Waddington and Low [6], so even this well know "success" of the theory is not that new. All of the models are simple and all produce patterns which look like the observed natural patterns. However, none of the biological models even allude to underlying mechanistic rules. This is not to say that CA models are not useful in biology. Indeed, in [2] we presented a number of examples of CA rules for biological systems which are tightly connected to the biology and provide some insight into the processes. I was impressed most by the applications of CA to fluid flows (particularly, the figure on page 380); in this case, the CA are often called lattice gases.

Chapter nine introduces reversible CA rules. These are automata whose state at time t can be run backwards to uniquely determine the state at time 0. Typically, reversible rules can be constructed from ordinary CA by making the state at time $t+1$ depend on both the state at time t and at $t-1$. This leads Wolfram to conservation laws and continuous models which segue into a subsection entitled *Ultimate models of the Universe.* This section is peppered with some of his favorite phrases: "My guess is ...", "I suspect that ...," "I believe that ..." typifying the "new kind of science" which requires no supporting evidence. The remainder of the chapter covers issues of fundamental physics such as gravity, space-time, and relativity. I cannot comment too much on this section and refer the reader to a lengthy review of the implications of this new physics [4].

Perception (according to Wolfram) is presented in Chapter ten. Here, he suggests models of visual (and other sensory) perception that appear to ignore fifty years of psychology, psychophysics, and neuroscience. The remainder of the chapter is devoted to a discussion of randomness and a rather lengthy (and dry to this reader) discussion of data compression.

The last two chapters represent the culmination of his theories of science which, as stated at the outset of this review, are (i) simple rules can generate complex behavior and (ii) all processes, human or natural, should all be regarded as computations (running some intrinsic program). The latter is the essence of what his the Principle of Computational Equivalence (PCE). In Chapter eleven, he provides the proof that the above-mentioned rule 110 is universal in the sense of a Turing Machine. That is, the rule can be used to emulate any finite computation. Curiously, this is the work of an "assistant" and not Wolfram, himself; and it has not been formally published. Here is one of several formulations of the PCE (pp 716-717): *almost all processes which are not obviously simple can be viewed as computations of* **equivalent** *sophistication.* This is a rather strong statement (a colleague likened this to the continuum hypothesis for computation.) What Wolfram says is that once you get beyond simple systems (presumably defined as the complement of complex systems which are never defined), all are in some sense equivalent to something like rule 110. This implies that however different the underlying structures, all complex systems are computationally equivalent. Furthermore, it implies that there are no systems which can be more complex. Thus, it would seem that Wolfram is finally getting around to at least the beginning of a definition of complexity: a complex system is equivalent to a universal computer. Wolfram argues that the traditional continuous mathematical models for natural systems cannot perform any more sophisticated computations than the discrete systems he has described in earlier chapters. He digresses into an interesting discussion of the intrinsic finiteness of CA rules in comparison to, say differential equations and concludes that even though ODEs run in continuous time, they will still not be able to produce, in finite time, a calculation requiring infinite time for a CA model.

One can ask what all of this means for those of us practicing traditional applied mathematics. In a sense, Wolfram's view is rather pessimistic. He argues that classic continuous methods can never model the behavior of systems with the kind of complexity seen in nature. He suggests, rather, that the simple CA models he describes should be used instead. Since the behavior of these systems (at least those he calls Class IV) can be understood only by running simulations of them, his ideas imply that analysis of equations is irrelevant to understanding natural phenomena of any complexity. I will leave the arguments of whether or not this is true to the philosophers. Until CA models can provide comparable insights into our understanding of the natural world that continuous systems do, I doubt very much that I will abandon the latter.

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