1. Consider the differential equation:

$$(D+1)^2(D^2+1)Dy = sin(3t) + e^{-t} + t$$

Using the method of undetermined coefficients, find the general form of solutions to this, but do not find the actual coefficients.

2. Consider

$$x' = \begin{pmatrix} -2 & 2 \\ -2 & 3 \end{pmatrix} x := Ax$$

(a) Sketch the phase-plane for this system and include the eigenvectors corresponding to the eigenvalues in your sketch.

(b) Suppose $x(0) = [1, c]$. Choose c so that $x(t) \to 0$ as $t \to \infty$.

(c) Use Fulmer's method to compute the matrix exponential $e^{tA} = \Phi(t)$.

(d) Use variation of parameters to find the general solution to

$$x' = Ax + \begin{pmatrix} e^t \\ e^{-2t} \end{pmatrix}$$

3. Classify all the phase portraits of the system:

$$\begin{aligned} x' &= x - ay \\ y' &= 3x + (1-a)y \end{aligned}$$

as the parameter a varies between $-\infty$ and ∞.

4. Find the real general solutions to the systems

$$x' = Ax$$

where

(a)

$$A = \begin{pmatrix} 2 & 5 \\ -1 & 0 \end{pmatrix}$$

(b)

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

5. Find the equilibrium points, determine their stability and sketch the phase-plane including the nullclines for the simple two species model below. You need only plot the behavior of the first quadrant. Sketch the trajectory starting at $x(0) = 1$ and $y(0) = 1/10$. What is the limit as $t \to \infty$ of $(x(t), y(t))$?

$$\begin{aligned} x' &= 2x(3 - x - 2y) \\ y' &= y(2 - x - y) \end{aligned}$$

6. Consider the following differential equation:

$$\frac{dx}{dt} = -y - x^2 - 2x + a$$
$$\frac{dy}{dt} = y + 2x$$

(a) Find all the equilibria of this system and their stability.

(b) For what values of a are there no equilibria?

(c) For what values of a are there stable equilibria?

(d) For what values of a are both equilibria unstable?

7. Suppose that a, b, c, d are all positive numbers and

$$x' = ax - by$$
$$y' = cx - dy$$

(a) Suppose that the slope of the $x-$nullcline is bigger than the slope of the $y-$nullcline. Prove that $(0, 0)$ is a saddle point.

(b) Suppose that the slope of the $y-$nullcline is greater than the slope of the $x-$nullcline and that $d > a$. Prove that $(0, 0)$ is asymptotically stable.

(c) Suppose $a < 0$. Prove $(0, 0)$ is asymptotically stable