

HOMEWORK 1

Due Sept 6

1. Prove that

$$\phi(t, x) = \frac{1}{\frac{1}{x} - t}$$

is a dynamical system.

2. Exercise 1 on page 5 in the lecture notes

3. Consider the ODE

$$\dot{x} = x^2, \quad x(0) = a$$

In the Cauchy-Peano existence theorem, solutions exist over an interval $[-b, b]$ where $b = \min\{\alpha, \beta/M\}$ where β, α, M are defined in the statement of the theorem. For this problem, choose a radius, β for your ball so as to maximize, b for given $a > 0$. Solve the ODE and compare your estimated interval of existence to the actual one.

4. Let $f(x) = Ax$ where A is a constant matrix. Show that each component of the n^{th} Picard iteration to any solution is a polynomial of degree at most n .