Homework # 5

1. Problems 5,6,9 chapter 10 (page 322-323)

2. Consider the extended Wilson-Cowan model:

$$\begin{aligned}
\tau_{11} z'_{11} &= -z_{11} + f_1(g_{11} z_{11} + g_{12} z_{12}) \\
\tau_{21} z'_{21} &= -z_{21} + f_1(g_{11} z_{11} + g_{12} z_{12}) \\
\tau_{12} z'_{12} &= -z_{12} + f_2(g_{21} z_{21} + g_{22} z_{22}) \\
\tau_{22} z'_{22} &= -z_{22} + f_2(g_{21} z_{21} + g_{22} z_{22})
\end{aligned}$$

   Prove that if $\tau_{11} = \tau_{21}$ and $\tau_{12} = \tau_{22}$ that all solutions to this ODE satisfy:

$$\lim_{t \to \infty} |z_{1j}(t) - z_{2j}(t)| = 0, \quad j = 1, 2$$

   and thus, they reduce to the WC equations.

3. Consider the two models for a scalar neural network with second and third order synapses:

$$\begin{aligned}
u'' + au' + bu &= bf(gu + I), \qquad (A) \\
u''' + au'' + bu' + cu &= cf(gu + I), \qquad (B)
\end{aligned}$$

   where a, b, c are all positive, and $f'(y)$ is monotone increasing (e.g. $f(u) = 1/(1 + \exp(-u))$) Can Model (A) undergo any Hopf Bifurcations to oscillations? How about Model (B) (Hint: use the Routh-Hurwitz criteria for these.) if your answer is yes to any of them, simulate an example. What is the sign of g in order to get oscillations. Prove that if $g < 0$, then each model has exactly one equilibrium point.

4. Problems 18 page 365

5. Page 399-401 number 1, 7,10