Homework #4

1. Problem 5, 6, 12, 23, 26,27,28 in Chapter 8 of the book

2. Tree Frog problem. Japanese tree frogs chirp rhythmically. When two of then are put together, they will sing in anti-phase so that they are a half period apart. When three of them are placed in the same cage, they sing in two distinct patterns that depend on the initial conditions, etc. In one pattern, 2 frogs sing in synchrony and the third sings out of phase with the others. In the second pattern, they sing in a ABCABCABC pattern where each frog is 1/3 cycle out of phase with the others. Your challenge is to construct a function $H(\phi)$ such that the model:

$$\theta_i' = 1 + \sum_{j \neq i} H(\theta_j - \theta_i)$$

satisfies the above constraints and such that the patterns described are asymptotically stable. I would first start with $N = 2$ and get some broad constraints on $H$; specifically, pairwise, the only attractor is antiphase. Then I would consider the second case of three frogs and look at the relative phases $\phi = \theta_2 - \theta_1$ and $\psi = \theta_3 - \theta_1$ so that there are just 2 equations and from these you can draw the phaseplane. Fixed points are locked states. I would try the simplest system: $H(\phi) = A \sin\phi + B \sin 2\phi + C \cos\phi$ and start with $C = 0$. Does $C$ affect the solutions? Is it even possible to get solutions when $C$ is nonzero? Which of your solutions are perturbed for nonzero $C$.

3. Consider the Fitzhugh-Nagumo model:

$$\begin{aligned} V' &= V(1-V)(V-a) - w + I \\ w' &= c(v - gw) \end{aligned}$$

Pick $I = 0.5, a = 0.1, g = 0.2, c = 0.05$ and get a limit cycle. Now, compute the phase resetting curve as an experimentalist would do. Inject a small square-wave of current at different times during the oscillation and measure the timing shift of the voltage. I would inject a pulse of width 0.5 and amplitude 0.1 and, for example, compute how the time at which $V(t)$ crosses 0 changes as the time of the pulse occurs. For these parameters a good initial condition on the limit cycle is $(0, .469721)$ and the period is about 37.71. So starting at these initial conditions, I would stimulate over a range between 0 and 37.71 and compute the time at which $V$ crosses 0 with positive slope. The PRC is just $T_0 - T_{hit}$ where $T_0$ is the unperturbed period and $T_{hit}$ is the time to cross. Do this for amplitude 0.02 and compare it. Try amplitude 0.1 again, but width, 0.1. Now, compute the adjoint. If you multiply the adjoint by the width*amplitude (area) of your pulse, you can compare it to the experimentally determined PRC. In XPP, to compute the adjoint, you just need to compute exactly one period and then click on Numerics Averaging Adjoint and then plot V vs t. Now, you can use the theory of averaging to compute the interaction function. We will assume diffusive coupling so that the coupling of say $V_2$ to $V_1$ is $V_2 - V_1$.

$$H(\phi) = \frac{1}{T}\int_0^T V^*(t)(V(t+\phi) - V(t))\ dt.$$

Compute this function numerically using the adjoint you computed. (In XPP, after computing the adjoint, click on Numerics Averaging Make H and then put V'-V in the V coupling and 0 in the w coupling and it will do it for you.

Finally, to complete this, couple two FN oscillators together and see if they always synchronize:

$$\begin{aligned} V_i' &= f(V_i, w_i) + \epsilon(V_k - V_i) \\ w_i' &= g(V_i, w_i) \end{aligned}$$

I would choose $\epsilon$ smallish.

Here is an XPP file for a single FN and also the coupled one.

```
v'=v*(1-v)*(v-a)+i-w + p(t-ton)
w'=c*(v-g*w)
par c=.05,a=.1,g=.2,i=.5
p(t)=amp*heav(t)*heav(wid-t)
@ total=1000,bound=100000
par amp=0,ton=1000
aux tstim=ton
par wid=.5
init v=0,w=.469721
@ dt=.01,total=50,maxstor=10000
@ xp=t,yp=v,xlo=0,xhi=50,ylo=-.5,yhi=.7
done
```

```
v1'=v1*(1-v1)*(v1-a)+i-w1+d*(v2-v1)
w1'=c*(v1-g*w1)+f*(w2-w1)
v2'=v2*(1-v2)*(v2-a)+i-w2+d*(v1-v2)
w2'=c*(v2-g*w2)+f*(w1-w2)
par c=.05,a=.1,g=.2,i=.5
par d=0,f=0
init v1=.3,w1=.1
@ total=200
done
```