HOMEWORK # 2
Due Feb 14

1. There were several questions in class, so you should do them. Specifically,

   (a) In the case of the ML model, we have a linearized matrix around (V_0, n_0) that has the form

   $$\begin{pmatrix} a & -b \\ c & -d \end{pmatrix}$$

   where b, c, d are positive and a could be positive or negative. Show that if the $V-$nullcline has a negative slope at the equilibrium, it is stable and $a < 0$. If it has a positive slope exceeding the slope of the $n-$nullcline it is a saddle, and if it has a positive slope that is less than that of the $n-$nullcline, it is a node and that $a > 0$.

   (b) For the ML model,

   $$I_{SS}(V) = g_l(V - E_l) + g_{Na}m_\infty(V)(V - E_{Na}) + g_K n_\infty(V)(V - EK)$$

   where the full model has the equations

   $$
   \begin{aligned}
   C\frac{dV}{dt} &= I - g_l(V - E_l) - g_{Na}m_\infty(V)(V - E_{Na}) - g_K n(V - E_K) \\
   \frac{dn}{dt} &= \frac{n_\infty(V) - n}{\tau_n(V)}
   \end{aligned}
   $$

   An equilibrium statisfies, $n = n_\infty(V)$ and $I = I_{SS}(V)$. Prove that the determinant of th linearization about the equilibium has the same sign as the derivative of I_{SS} at the equilibrium

   (c) Consider the ode for z a complex valued variable:

   $$z' = z(p + i + (a + ib)|z|^2 - |z|^4)$$

   where p is the bifurcation parameter and a, b are real numbers. Compute the bifurcation diagram for this by first writing $z = re^{i\theta}$ and then finding solutions of the form $r = r_0$ and $\theta = \omega t$ where for $r_0 > 0$ this corresponds to a limit cycle. Assess stability etc

   (d) Consider the Takens normal form for the double zero eigienvalue case:

   $$x' = y + bx - x^2, \qquad y' = a + x^2$$

   Find the equilibria and their stability. Find the vaules of (a, b) where there are zero eigenvalues at equilibria (nodes or folds) and where there are Hopf bifurcations (there are pure imaginary eigenvalues at one of the equilibria). Fix $a = -1$ and estimate the value of b (numerically!) where there is a homoclinic bifurcation. Sketch a bifurcation diagram as b varies between -3 and 0 for $a = -1$. You can do this qualitatively

2. Book problems: Chapter 3 (page71-74) 4,6,12,13; Chapter 4 (page 96 or so) 2,8,9,11

Here is a quick and dirty tutorial on computing the bifurcation diagram for the Butera model that I discussed briefly in class. Here is the code in XPP. You can try his in MatLab using MatCont, I suspect, but I don't know how.

```
# butera and smith model using NaP
par cm=21,i=0
xinf(v,vt,sig)=1/(1+exp((v-vt)/sig))
taux(v,vt,sig,tau)=tau/cosh((v-vt)/(2*sig))
# leak
il=gl*(v-el)
par gl=2.8,el=-65
# fast sodium --  h=1-n
minf(v)=xinf(v,-34,-5)
ina=gna*minf(v)^3*(1-n)*(v-ena)
par gna=0,ena=50
# delayed rectifier
ninf(v)=xinf(v,-29,-4)
taun(v)=taux(v,-29,-4,10)
ik=gk*n^4*(v-ek)
par gk=0,ek=-85
# NaP
mninf(v)=xinf(v,-40,-6)
hinf(v)=xinf(v,-48,6)
tauh(v)=taux(v,-48,6,taubar)
par gnap=2.8,taubar=10000
inap=gnap*mninf(v)*h*(v-ena)
v' = (i-il-ina-ik-inap)/cm
n'=(ninf(v)-n)/taun(v)
h'=(hinf(v)-h)/tauh(v)
@ total=40000,dt=1,meth=cvode,maxstor=100000
@ tol=1e-8,atol=1e-8
@ xlo=-85,xhi=55,ylo=-.1,yhi=1.1
@ xp=v,yp=h,nmesh=100
done
```

This shows the full model, but we will simplify it by eliminating the fast sodium and potassium $(g_{na} = g_k = 0)$

1. Run this model in XPP. It is set up for the phaseplane view with the fast channels turned off and the phaseplane is the V, h plane. The solver used is a stiff solver so I can take big jumps without losing accuracy as it is set up for 40 seconds!. Get the nullclines by Nullcline-New (NN). Integrate the equations to steady state by Initialcond Go and then Initialcond Last (IG IL). Do the last thing a few times to make sure you are at a steady state. The figure below shows the phaseplane for several values of el.

2. Bring up AUTO, File Auto (FA). The rest of this will all be in the AUTO window. Click Axes Hi-Lo. Change Main Parm to el (it is currently set to cm) . Change xmin to -70, ymin to -80, xmax to -50, ymax to 55. Click OK when done. Click Numerics. Change Dsmax to 0.1, Par Max to -70, Par Min to -50 and click OK.

3. Get the steady states: Click Run Steady state. You should see a line across the screen that changes from red to black to red. Red is stable equilibria and black is unstable.

4. Get the periodic orbits. Click on Grab. (A cross will appear in the screen. Use the left/right arrow keys to move along the diagram. Info is at the bottom of the screen. You want to get to the point labeled "3". It is easier to tap the TAB key that moves only to labeled points. (You can use the mouse to get close to it as well). If you did it right it should say HB at the bottom. You do not want to get the first HB point (labeled 2), you want the second!! Once the cross is on the labeled point 3, click Enter. Now click Run Periodic and a curve of green dots will emerge. When the parameter value is close to point 2, click the abort key to stop things. If all worked out (and it probably wont!) you will see the figure below. If you want to save the pictur, click File Postscript and give it a nice name

5. Exit XPP. There are lots of other things you can do, but this is enough. In the XPP window, click File Quit Yes