

Consider a weakly nonlinear dendrite

$$0 = f(V) + \frac{d^2V}{dx^2}$$

at steady state subject to injected current, $dV/dx(0) = a$ at $x = 0$ and sealed ends, $dV/dx(L) = 0$ at $x = L$. Assume $f(V)$ has 3 roots and the $(V, dV/dx)$ phase plane is as illustrated. Picking a to be positive and L very large, but not infinite, analyze how many solutions to this nonlinear boundary-value problem there are as a varies. Note the blue curves are the stable manifolds of the saddle points and solutions go to the fixed points as $x \rightarrow \infty$. The fixed points are $dV/dx = 0$ and V at any of the three circles. The red lines correspond to setting dV/dx at certain values of a . I have drawn a couple of solutions to the ode with $dV/dx(0) = a$ and $V(0)$ at different values. You can see that these trajectories can satisfy both end conditions for a particular L . Thus note that only trajectories starting on the red lines and crossing $dV/dx = 0$ are candidate solutions.

