



Peristaltic Transport as the Travelling Deformation Waves

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Geometry approach to the theoretical and experimental investigations of peristaltic waves based on the travelling deformation waves and wave mass transfer theory (Dobrolyubov, 1991) is presented. The theory of travelling deformation waves is employed to determine uniformed expressions for mass transfer capability parameters of peristalsis. Slow (quasi-static) wave motion is considered which permits not to take into account dynamic phenomena.

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Introduction

Peristaltic transport of viscous liquids and mixtures is the most important biomechanical instrument of the digestive system of humans and animals. Being a reliable “valveless pump”, insensitive to solid inclusions in the medium being transported, the peristaltic mechanism provides for extremely slow, discontinuous (impulsive) forward motion of the bowels content, accompanied at the same time by intensive mixing. The peristaltic method of transporting the contents of the digestive tract of animals and humans can be considered a purely volumetric, rather than a dynamic process.

In this article, we will discuss the travelling deformation waves, which are used in intestines. In intestines, the peristaltic transport is characterized by slow waves of small amplitude. In the other parts of the digestive system, the mechanism is quite different. The considered

peristaltic single wave is the slowly travelling widened (convex) or narrowed (concave) segment on the thin wall elastic tube filled with viscous incompressible liquid. Figure 1 shows both the possible kinds of single peristaltic waves—the widened (a) and narrowed (b).

We shall analyse qualitatively and quantitatively the transport capabilities of such waves and give the uniform mathematical expressions for the flow rates and velocities in arbitrary cross-section of such tubes. We will give expressions for the instantaneous and time interval wave mass transfer, i.e. for quantity of mass (or volume) of liquid, which is transferred through the cross-section by the peristaltic wave travelling along the tube. It will be shown that the wave mass transfer process is a renewal (relay) one when the transported mass being constant in quantity is permanently renewing in the wave crest during the wave motion. It will be given proper unified expressions for evaluations of the rate of the renewal process in the waves of different forms. We shall accept as assumption, which was proved experimentally, that peristaltic

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(A.I. Dobrolyubov).

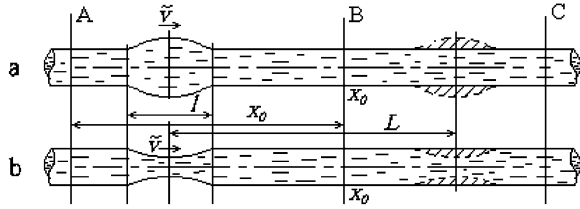


FIG. 1. Peristaltic tubes: (a) convex travelling peristaltic wave and (b) concave travelling peristaltic wave.

single wave generates a local motion of the viscous liquid in the wave segment of the deformable tube (segments l in Fig. 1) and the liquid in the other (out of wave) segments of the peristaltic tube is at rest. This assumption is permissible if the value of the volume of the crest or the trough is permanent and if the cross-sections before and after the wave are equal to each other [for example, cross-sections at A, B, C (Fig. 1) must be the same before and after the wave passage]. In this case, the volume between two arbitrary cross-sections A and B or A and C (Fig. 1) remains constant during wave motion. This means that the viscous liquid in the out of wave segments (for example, segment BC) is motionless, i.e. the flow rate through any cross-section of the segment is equal to zero, and the peristaltic tube can be regarded as closed at its out of waves cross-sections. This is proved by experiments with peristalsis of the viscous liquid (Dobrolyubov, 1996).

We will count that all geometric parameters of the peristaltic tube and the wave and the velocity of the wave are given. We will use the following mathematical symbols for the parameters: S_0 is the value of cross-section area of the peristaltic tube in the undeformed (neutral) state, S_x the same for the cross-section in the arbitrary point of the deformed wave segment of the tube, \tilde{v} the velocity of the wave, \bar{v}_x the averaged velocity of the liquid in the cross-section, l the length of the wave (of deformed segment of the tube), $\gamma = \text{const}$, the volume density of the liquid, L the distance of the wave's travel along the tube, q the flow rate through the cross-section x , Q the mass transfer of the wave, $V_0 = lS_0$, the volume of the undeformed tube of length l ; \tilde{V} the volume of all the wave of length l , $\Delta\tilde{V} = \tilde{V} - V_0$, the volume contained only in the bulge of the wave,

and $\Delta l = \Delta\tilde{V}/S_0$, the equivalent length of the bulge in the tube.

Transport Capability of the Convex Peristaltic Wave

First, consider the convex peristaltic wave of invariable geometric form [Fig. 2(a)]. Consider two successive positions of the wave, which have displaced from left to right along the x -axis by a small distance δx . Note that the surface f of the leading edge of the travelling wave being displaced by the distance δx sweeps out some volume δV , which is marked by “+”, and the surface b of the trailing edge sweeps out the same volume δV , which is marked by “-”. In other words, the trailing edge of the wave displaces some volume δV of the liquid and the leading edge f sucks up the equal volume δV . Therefore, the liquid of δV volume flows from the evacuated space (-), which is near the trailing edge of the wave, to the vacant space (+), which is near the leading edge. This means that the liquid inside the convex wave segment l is moved in the same direction as the wave (forward wave mass-transfer). So, this means that the liquid motion generated by the travelling *convex* wave exists only inside the wave segment, i.e. between the trailing and leading edges of the wave. And liquid remains at rest in other segments of the peristaltic tube (outside the wave segment l of the tube). (Here, we understand statistically the rest of the liquid when liquid particles can move

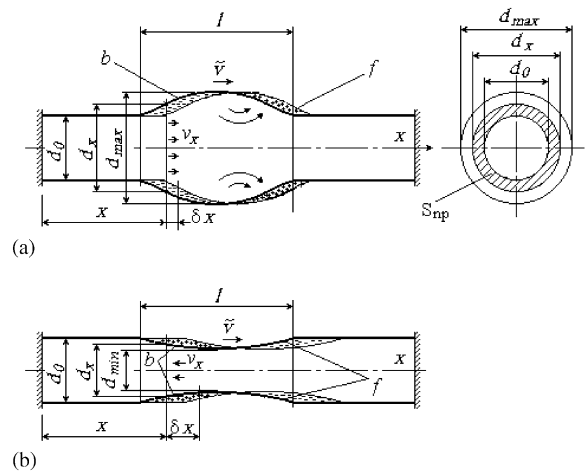


FIG. 2. Peristaltic waves as travelling processes of displacement and suction of liquid.

chaotically, but the rate of flow in any cross-section is zero). So we confirm the said conclusion that the motion of the liquid in the peristaltic tube is a local one. This means that the liquid particles which are in the wave segment at the moment are moved, and the particles which are out of the wave segments are at rest. Taking into account this fact and that the liquid in the peristaltic tube is rather viscous (for example, as in the digestive systems of humans or animals), we may regard the peristaltic tube as being closed at both ends by the impermeable partitions. (In Fig. 1 such, imaginary or real, partitions may be inserted, for example *A, B, C* cross-sections of the tube). The local method of peristaltic transportation (the liquid is in motion in the short segment while it is at rest in the others segments) is important with respect to the reduction of the required power. The peristaltic transport generates a very slow motion of the liquid (the slowness is the main required biomechanical and physiologic parameter of peristaltic transport in the digestive tract of human and animal).

Now we will find the magnitude of flow rate (mass-transfer) and the velocity of the liquid in the convex peristaltic tube [Fig. 2(a)]. The wave of constant form travels with a velocity \tilde{v} and during the small time δt is displaced on the distance $\delta x = \tilde{v}\delta t$ along the x -axis. During this time, as we said, the trailing edge b (this is cone-like curve surface) displaces the volume δV of the liquid (marked with signs “-”). The leading wave edge f , which also has the cone-like form (not obligatory of the same form as of the trailing edge), sucks in the liquid of the same volume δV (marked with signs “+”). The value δV can be found by the geometric method if all dimensions of the wave and the tube are known. It is known from geometry, that the volume described (swept out) by a moving curve surface is equal to the area S_{pr} of projection of this surface on the perpendicular plane to the direction of motion, multiplied by the value of the displacement δx of the moving surface. For our wave [Fig. 2(a)], the area of the projection of the trailing edge is $S_{pr} = S_x - S_0 = \pi d_x^2/4 - \pi d_0^2/4 = \pi(d_x^2 - d_0^2)/4$ [this area is cross-hatched in the right part of the Fig. 2(a)] and the displacement is δx . So the swept out by the

trailing edge b during δt volume is $\delta V = \delta x(S_x - S_0) = \delta x\pi(d_x^2 - d_0^2)/4$ (the same volume δV is swept out by the leading edge f of the wave). This volume is the volume of the liquid, which passes during time δt through fixed arbitrary section S_x of the wave segment of the tube. Therefore, the volume which passes through this section during time unit, i.e. the volume flux rate $q_x(m^3/s)$ is

$$q_x = \delta V/\delta t = (\delta x/\delta t)(S_x - S_0) = \tilde{v}(S_x - S_0). \quad (1)$$

The mass flow rate is $q_x^m = q\gamma$ (kg s^{-1}).

The average velocity \bar{v}_x of the fluid particles, which cross the section S_x of the peristaltic wave is

$$\bar{v}_x = q_x/S_x = \tilde{v}(S_x - S_0)/S_x. \quad (2)$$

If the whole peristaltic wave crosses the fixed section S_x in time T , then the volume Q_x of the fluid, which crosses S_x is

$$Q_x = \int_0^T q_x dt = \int_0^T \tilde{v}(S_x - S_0) dt. \quad (3)$$

Taking into account that $\tilde{v} = dx/dt$ and $l = \tilde{v}T$, we receive

$$\begin{aligned} Q_x &= \int_0^l q_x dx/\tilde{v} = \int_0^l (S_x - S_0) dx \\ &= \int_0^l S_x dx - \int_0^l S_0 dx = \tilde{V} - V_0 = \Delta\tilde{V}, \quad (4) \end{aligned}$$

where \tilde{V} is the volume of the wave segment l , and V_0 is the same volume without the volume of the crest, which is cross-hatched in Fig. 1(a). So the “volume-transfer” $\Delta\tilde{V}$ of the convex peristaltic wave quantitatively is equal to the volume of the wave’s crest. The mass-transfer of the wave is equal to the quantity of mass contained in this volume.

Transport Capability of the Concave Peristaltic Wave

Similar “geometrical approach”, which we have used for the analysis of the convex wave, permits to establish that the concave single peristaltic wave also generates the motion of the liquid inside the wave segment only and in

the out of the wave segments the liquid remains at rest. The difference of the concave wave's motion from the convex one is that in the concave wave segment the liquid is moved in opposite to the wave's direction. The explanation of this fact is that in the concave wave, the leading edge f displaces the liquid [Fig. 2(b)] and the trailing edge b sucks in the liquid. Therefore, the liquid is moved from the evacuated region (marked by “-”) to the region of suction (marked “+”), i.e. from the leading to the trailing edge [this is shown in Fig. 2(b) by arrow v_x].

The following argumentation can also prove, for concave wave, the conclusion about the opposite directions of wave and liquid motions. Imagine that at the initial moment a concave wave was to the left of the fixed section B [Fig. 1(b)]. The volume contained in the left part of the tube (between A and B) was equal to V_l , and in the right part of the tube (between sections B and C) was equal to V_r . It is obvious that after the run of the wave to the right position (drawn with the dotted line), the left part of the tube will contain the volume $V_l + \Delta\tilde{V}$ and right part will contain $V_r - \Delta\tilde{V}$, where $\Delta\tilde{V}$ is the volume of the trough of the wave. This means that the liquid has flowed to the left, i.e. against the direction of the wave motion.

For the concave peristaltic wave, formulas (1)–(4) for the flow rate and velocity remain the same as for the convex wave, but here $S_x < S_0$ and therefore the values calculated become negative, which means the opposite directions of velocity or flux relative to the direction of the wave motion.

Physical Features of the Wave Peristalsis

It follows from the above discussion that as the convex peristaltic waves the concave ones transfer with their velocity the liquid mass $\Delta m = \gamma\Delta\tilde{V}$, which is equal to the mass of the liquid contained in the volume $\Delta\tilde{V}$ of the crest or trough (in the latter case we shall regard the value Δm as negative). We shall call the mass Δm as the *mass content of wave*. For a convex wave it is positive, for a concave one it is negative. For a convex wave, the mass contained in it is transferred with the wave's velocity \tilde{v} . So the statement holds true that the convex wave, which

has travelled by the distance L , has transferred for this distance the mass Δm [Fig. 1(a)].

From the point of view of the mass-transfer conception, the travelling wave can be regarded as a transport mechanism, which can transport the mass Δm with the wave velocity \tilde{v} . When the wave is formed at one end of the tube, it is as if the quantity of mass $\Delta m = \gamma\Delta\tilde{V}$ is loaded into the wave. When the wave travels it is as if this quantity Δm of the liquid is carried with the wave. Then when the wave collapses at the other end, it is as if the same quantity is unloaded. So the travelling waves play a typical role of a mass-transfer mean. The difference of wave mass-transfer from an ordinary transport means (in container) is that at the arrival the set of particles in the wave is different from the beginning (Dobrolyubov, 1991).

This occurs because of the continuous process of renewing of mass in the travelling wave: during any time interval some quantity of particles enter the travelling wave and the same quantity of particles leave the wave. For the time of the wave travelling at a large distance L (Fig. 1), all particles contained initially in the wave can leave the wave and others can occupy their places. In that case, it is not those particles of liquid which were at the start position which will arrive at the finish but completely new ones, i.e. the full renewal of liquid takes place in the wave. The stepping (discrete-wave) motions of liquid particles accompany such renovation of liquid. In the case of the concave wave travelled by a distance L [Fig. 1(b)], the result of mass-transfer can be interpreted as a withdrawing of mass Δm from the finish region of the wave and an arrival of the same quantity of mass at the start region.

Now, we find some kinematic characteristics and estimate the quantitative rate of renewal processes in the waves of different forms.

(1) Minimal distance L_R in which full renewal of liquid in the wave segment l can take place. Consider two neighbor segments of the peristaltic tube filled with incompressible viscous liquid: the wave segment of length l and volume \tilde{V} , and the previous undeformed cylindrical segment of cross-section S_0 and length L_R , the volume of the undeformed segment being also \tilde{V} (We again can consider the tube as closed at its ends A and B).

The length L_R is the minimal distance of wave travel in course of which the full renewal of liquid in the wave segment l can occur. In other words, L_R is the distance of travel of the wave when the volume \tilde{V} of the liquid can enter into the travelling wave segment l and the same volume of the liquid can leave the wave segment. The above said does not mean that for the time $\tilde{t} = L_R/\tilde{v}$ of running of the wave all particles really will have left the wave segment l . There may be cases when some particles do not leave the wave segment l during time \tilde{t} of a wave travelled by a distance L_R , but this will occur for the account of others particles of this segment which may enter and leave the wave segment during the wave travelling by L_R distance. Therefore, the value L_R is the way of the travelling wave when all particles of the wave segment in average will be renewed by new ones, that is the way in course of which the volume \tilde{V} of a new liquid enters the wave segment and the same volume of liquid leaves the segment. From this follows that

$$L_R = \frac{\tilde{V}}{S_0} = \frac{V_0 + \Delta\tilde{V}}{S_0} = \frac{V_0}{S_0} + \frac{\Delta\tilde{V}}{S_0} = l + \Delta l. \quad (5)$$

(2) The average time \tilde{t} of being of arbitrary particle in the travelling wave segment, i.e. the averaged time of motion of the particle during one run of the wave, is equal to the value L_R of the travelled distance divided by the value \tilde{v} of wave velocity

$$\tilde{t} = \frac{L_R}{\tilde{v}} = \frac{\tilde{V}}{S_0\tilde{v}}. \quad (6)$$

(3) The average step $\Delta\bar{x}$ is the average distance which the liquid particles traverse during one run of the wave. We can find it as a sum (integral) of elementary average displacements $\delta(\Delta x) = \tilde{v}\delta t$ for time \tilde{t} . Taking into account that $dt = dx/\tilde{v}$ and eqn (2)

$$\begin{aligned} \Delta\bar{x} &= \int_0^{\tilde{t}} \tilde{v} dt = \int_0^{L_R} \tilde{v} \frac{S_x - S_0}{S_x} \frac{dx}{\tilde{v}} \\ &= \int_0^{L_R} \frac{S_x - S_0}{S_x} dx = \int_0^{L_R} dx - \int_0^{L_R} \frac{S_0}{S_x} dx \\ &= L_R - \int_0^{L_R} \frac{dV_0}{S_x} = L_R - l = \Delta l. \end{aligned} \quad (7)$$

This means that the average value of free path of the liquid particles in the peristaltic wave coincides with the length of the undeformed segment of the tube, the volume of which is equal to the volume of the crest of the peristaltic wave. Taking to account that $\tilde{V} = L_R S_0$ and $V_0 = l S_0$ eqn (7) can be transformed into the form

$$\Delta\bar{x} = \frac{(\tilde{V} - V_0)}{S_0} = \frac{\Delta\tilde{V}}{S_0}. \quad (8)$$

(4) The average velocity \bar{v}_x of the liquid particles, which are at the moment in the wave segment l of the peristaltic wave, is equal to the average displacements $\Delta\bar{x}$ of particles divided by the average time \tilde{t} of being of the particles in the wave segment

$$\bar{v}_x = \frac{\Delta\bar{x}}{\tilde{t}} = \frac{\tilde{v}(L_R - l)}{L_R} = \tilde{v} \left(1 - \frac{l}{L_R}\right). \quad (9)$$

Taking into account that $\tilde{v} = L_R S_0$ and $V_0 = l S_0$ the value \bar{v}_x can be transformed into the form

$$\bar{v}_x = \tilde{v} \frac{\tilde{V} - V_0}{\tilde{V}}. \quad (10)$$

(5) The average velocity $\bar{\bar{v}}_x$ of all liquid contained in the peristaltic tube of length L during the time of run of the wave along all this tube is equal to

$$\bar{\bar{v}}_x = \tilde{v} \frac{\Delta\bar{x}}{L} = \tilde{v} \frac{L - l}{L} = \tilde{v} \frac{\Delta\tilde{V}}{V_L}, \quad (11)$$

where $V_L = S_0 L$ is the volume of all liquid contained in the tube of length L .

All equations (5–11) are true not only for convex peristaltic wave, but also for concave ones. In the case of concave wave $S_x < S_0$ and from the above equations it follows that the values $\Delta\bar{x}$, \bar{v}_x , $\bar{\bar{v}}_x$ become negative. This means that the backward transfer of the liquid by the concave peristaltic wave takes place.

The Peristaltic Waves of Complex Forms

From the above conclusions, we know that the convex and concave travelling waves transfer mass in directions opposite to each other. The complex (combined) peristaltic waves consist of

sequences of convex and concave waves (of crests and troughs). So the complex waves transport the liquid along the tube in the sameway in the forward and backward directions. The magnitude and direction of the peristaltic transportation depend on the geometric parameters and on velocity of the travelling waves but are independent of physical properties of the liquid, provided that the conditions of peristalsis are respected.

It follows from this that, for example, symmetric peristaltic waves for which the volume of crests is equal to the volume of troughs ($\Delta\tilde{V}_+ = \Delta\tilde{V}_-$) [Fig. 3(a)] do not transport the mass along the peristaltic tube. They only oscillate it in longitudinal and transverse directions, because the positive mass-transfer of the convex part of the wave is equal to the negative mass-transfer of the concave part. In case of asymmetry waves (when $\Delta\tilde{V}_+ \neq \Delta\tilde{V}_-$), the result of mass-transfer can be different and it depends on differences between the volumes of the crests and troughs. If for asymmetry wave $\sum(\Delta\tilde{V}_+) > \sum(\Delta\tilde{V}_-)$ [the volume of the crests is larger than the volume of the troughs; these are mainly convex waves, [Fig. 3(b)], the result mass-transfer is directed forward (in the direction of the wave motion) and $\sum Q_x = (\sum(\Delta\tilde{V}_+) - \sum(\Delta\tilde{V}_-)) > 0$. In the case of mainly concave waves, when $\sum(\Delta\tilde{V}_+) < \sum(\Delta\tilde{V}_-)$, Fig. 3(c), the result mass-transfer is $\sum Q_x < 0$, i.e. it is directed backward (against the wave velocity \tilde{v}). It is obvious that

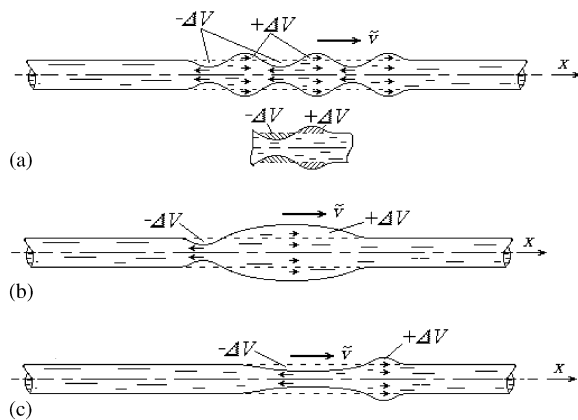


FIG. 3. Peristaltic waves with compound shapes: (a) symmetry shape of waves (the volume of bulges is equal to the volume of waves); (b) predomination of the bulge; and (c) predomination of the wave.

the single peristaltic wave (Fig. 1) is an asymmetric one and it accomplishes, as was shown, directional stepping transfer of the liquid.

About Correspondence of the Described Results to the Peristaltic Mechanism in Digestive Tracts in Animals and Humans

The digestive tract (intestine) of animals and humans is a long (8–10 m for humans) tube with thin elastic walls filled with viscous liquid and many times bent. The peristaltic waves, i.e. the convex and concave segments, periodically travel along these walls. These travelling along the elastic tube wave-like swellings present a very simple hydrodynamic mechanism which solves, from mechanics and engineering point of view, very difficult problems—to provide very slow (in average about 1.5 cm/min) reliable transport of viscous liquid in which hard inclusions are possible. Let us consider in what degree the above-described geometrical and mathematical models of peristalsis correspond to the peristaltic mechanism in living organisms.

Analyses of literature on biomechanics of peristalsis permit to draw the following conclusions about existing approaches to biomechanics of peristaltic transport.

- Peristaltic waves have convex–concave form and in the normal conditions of the digestive system can move along the intestine in both directions, depending on the physiological situation in the digestive tract. This, in accordance with our conclusions about wave mass transfer, indicates that peristaltic waves on the intestine walls are mainly convex ones [convexities predominate over concavities, [Fig. 3(b)] which transport the liquid in the direction of wave motion. Normally, the velocity of peristaltic waves is about $1\text{--}2\text{ cm s}^{-1}$ (Sturkie, 1981).

- Each single peristaltic wave (the travelling wave deformed section of intestine) has a rather small length in comparison to the length of the neighbor undeformed segments of the intestine. It provides that the liquid is moved in the wave segment only and is at rest in the other (undeformed) segments of the intestine. From that can be drawn a conclusion about the stepping (discrete-wave) character of the transported

liquid: being in the wave segment the particles are moving and being outside the wave segments they are at rest. Such transport of liquid, as we described, is local.

- Geometric form of peristaltic waves on the intestine wall is usually complex, i.e. they include contractions as well as expansions of the intestine, and the expansions (crests) of the waves are larger in their volume than the contractions (troughs). That agrees with our conclusion about the forward mass-transfer by mainly convex waves.

- Muscular layer in the walls of the intestine contains ring (circular) muscles, which are capable to form the travelling contractions of the intestine, and longitudinal ones which are capable to form travelling expansions on the intestine (Schmidt & Thews, 1983). Thus, convex and concave peristaltic waves are formed and moved by the muscular layers in the walls of the intestine.

- The single peristaltic wave can arise in any segment of the intestine owing to the deformation of its walls and being travelled by some distance it may disappear also in any other place of the intestine when the walls of intestine have lost their deformation and go to the normal undeformed state. The forming, travelling, and disappearing of the waves are controlled by the nervous structures.

The peristaltic transport mechanism is often described from others (not travelling deformation wave) approach (Sturkie, 1981; Encyclopedia and Dictionary of Medicine, Nursing and Allied Health, 1987; Eskert *et al.*, 1988). “Peristalsis is the traveling wave of contraction of ring muscles which follows the segment of feebleness of muscles. As a result the food’s lump is traveled along the intestine” (Eskert *et al.*, 1988). “Peristalsis forces the food into and through the intestine for further digestion until the food waste finally reaches the rectum” (Encyclopedia and Dictionary of Medicine, Nursing and Allied Health, 1987). Such a very popular explanation of the peristaltic transport does not describe quantitatively this process. It is only a qualitative interpretation of the process.

In concave waves, liquids or lumps move in the opposite direction of the wave. Convex waves also generate motion. At the same time waves reduce the motion of the particles. In both cases (concave and convex wave), the particles accomplish the discrete-wave (stepping) motions. In case of complex profile of the peristaltic waves, the particles of liquid accomplish difference in value displacements in opposite directions along the intestine. It permits to mix the mixture inside the intestine. Thus to the author’s mind, the widespread explanation of peristaltic wave mechanism as travelling ring contraction which drives (drives out) the liquid before it is incorrect.

Conclusion

The travelling deformation waves, which we have used for the analysis of the biological wave’s method of transportation in the digestive system of man and animal, is rather a general biological instrument. This instrument is used as driving mechanisms of wave locomotion of some terrestrial soft-bodied animals as the caterpillar, earthworm, snake and snail. So the travelling deformation waves and wave mass-transfer theory is used for designing wave devices for different engineering functions (Dobrolyubov, 1986).

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