1. Problem 3.1 #1 page 128 in the book
2. Let $k(x) = (1/2)\exp(-|x|)$. Consider the integral equation:

$$u(x) = \int_{-\infty}^{\infty} k(x-y)F(u(y))\ dy$$

Show that if $u(x)$ is a solution to this, then

$$-u_{xx} + u(x) = F(u(x))$$

Suppose that $g(u) = -u + F(u)$ has three zeros, say, $u = 0, a, 1$ and that $0 < a < 1$. Also assume that $g'(0) < 0$, $g'(a) > 0$ and $g'(1) < 0$ so that g looks roughly like the cubic function $u(1-u)(u-1)$. Write the above ODE as a phase plane problem

$$u_x = v \qquad v_x = -g(u)$$

and use this to sketch possble bounded solutions $u(x)$ on the whole line.

3. Consider the integral equation

$$u(t) = \int_0^{\infty} k(s)u(t-s)\ ds.$$

where

$$\int_0^{\infty} |k(s)|\ ds < \infty.$$

Show that $u(t) = \exp(\lambda t)$ is a solution to this linear equation if λ satisfies certain criteria. Find such λ if

$$k(s) = A\exp(-\beta s)\cos(\alpha s)$$

where β, α are positive.