1. Problem 3.1 #1 page 128 in the book

2. Let \( k(x) = \frac{1}{2} \exp(-|x|) \). Consider the integral equation:

\[
u(x) = \int_{-\infty}^{\infty} k(x - y)F(u(y)) \, dy
\]

Show that if \( u(x) \) is a solution to this, then

\[-u_{xx} + u(x) = F(u(x))\]

Suppose that \( g(u) = -u + F(u) \) has three zeros, say, \( u = 0, a, 1 \) and that \( 0 < a < 1 \). Also assume that \( g'(0) < 0, g'(a) > 0 \) and \( g'(1) < 0 \) so that \( g \) looks roughly like the cubic function \( u(1-u)(u-1) \). Write the above ODE as a phase plane problem

\[
u_x = v \quad v_x = -g(u)
\]

and use this to sketch possible bounded solutions \( u(x) \) on the whole line.

3. Consider the integral equation

\[
u(t) = \int_{0}^{\infty} k(s)u(t - s) \, ds.
\]

where

\[
\int_{0}^{\infty} |k(s)| \, ds < \infty.
\]

Show that \( u(t) = \exp(\lambda t) \) is a solution to this linear equation if \( \lambda \) satisfies certain criteria. Find such \( \lambda \) if

\[
k(s) = A \exp(-\beta s) \cos(\alpha s)
\]

where \( \beta, \alpha \) are positive.