1. If A is Hermitian and $< x, x >= 1$ show that

$$\lambda_{max} \geq < Ax, x > \geq \lambda_{min}$$

2. Find the eigenvalues of the general circulant matrix

$$A = \begin{pmatrix} a_0 & a_1 & \cdots & a_{n-1} \\ a_{n-1} & a_0 & \cdots & a_{n-2} \\ \vdots & \ddots & \ddots & \vdots \\ a_1 & a_2 & \cdots & a_0 \end{pmatrix}$$

3. Suppose that for the matrix $A = (a_{ij})$

$$|a_{ii}| > \sum_{j=1, j \neq i}^{n} |a_{ij}|$$

   for all i. Prove A is invertible.

4. A general resistive/capacitive linear circuit has the following equations:

$$C_i \frac{dV_i}{dt} = I_i + g_i(E_i - V_i) + \sum_j g_{ij}(V_j - V_i)$$

   where $C_i, g_i > 0$, $g_{ij} \geq 0$, and I_i, E_i are real constants. Note that the units of g are 1/ohms. Prove that (i) there is a unique equilibrium solution to this differential equation and (ii) that it is asymptotically stable.

5. A matrix A is called positive definite if A is Hermitian and if $< Ax, x >> 0$ for all $x \neq 0$. Prove

   (a) All eigenvalues of A are positive.

   (b) Let A, B be positive definite. Consider the eigenvalue problem:

$$\left(\lambda^2 I + \lambda A + B\right) v = 0.$$

   Show that all solutions to this satisfy $\mathrm{Re}(\lambda) < 0$.