HOMEWORK 8
Due Nov 29

1. Consider the ode:

$$\begin{aligned} x' &= -x^3 + 3xy^2 \\ y' &= -3x^2y + y^3 \end{aligned}$$

   Compute the index of $(0,0)$.

2. Sketch the phase portraits for each of the following planar systems. If useful, find the nullclines as well. If they are Hamiltonian (like in the previous exercises) use this fact to help you plot the portraits. If they have limit cycles, make sure you plot them too!

   (a)

$$\begin{aligned} x' &= x(4 - x - 2y) \\ y' &= y(3 - x - y) \end{aligned}$$

   (b)

$$\begin{aligned} x' &= -x + \tanh(5x - 8y) \\ y' &= -y + \tanh(x) \end{aligned}$$

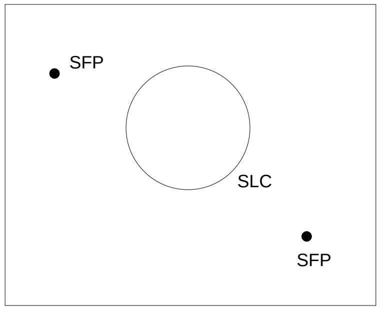
   (c)

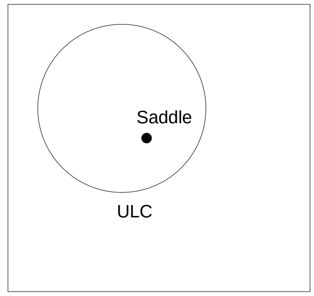
$$\begin{aligned} x' &= \frac{1}{4}y^3 - x \\ y' &= x(1 - x) + y \end{aligned}$$

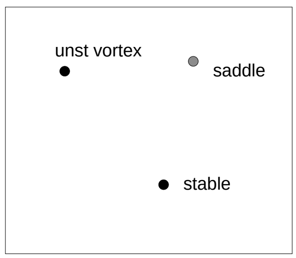
3. Suppose that $f(x,y)$ and $g(x,y)$ are continuously differentiable in the plane. Suppose that $f_y g_x > 0$ for all x, y in the plane. Prove that there are no periodic solutions to $x' = f(x,y)$, $y' = g(x,y)$. (Hints: WLOG assume $f_y > 0, g_x > 0$. Note that if $x(t), y(t)$ oscillate, then they must have both local minima and local maxima.)

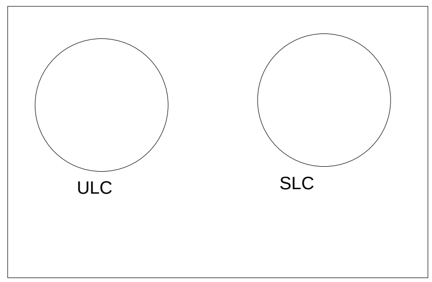
4. Fun exercise! For the figures below, determine which phase portraits are possible for a continuous planar system and fill them in. If they are impossible, add whatever fixed points or limit cycles that you need to make a valid portrait. SFP is stable fixed point, SLC is stable limit cycle, ULC is unstable limit cycle.

5. Boring exercise. Do the series hand out exercises:1,2 page 3; 1,3 page 6.


SFP
SLC
SFP


Saddle
ULC


unst vortex
saddle
stable


ULC
SLC