Homework # 5

1. Find a change of variables to put the following matrix into real Jordan form:
$$A = \begin{bmatrix} 1.5 & 2 & -0.5 & 0 \\ -2 & 1.5 & 0 * -0.5 & \\ 0.5 & 0 & 0.5 & 2 \\ 0 & 0.5 & -2 & 0.5 \end{bmatrix}$$

2. Find a 2×2 self-adjoint positive-definite matrix that is complex.

3. Show that the matrix $A \in M_n(R)$ consisting of all 1's is positive semi-definite, but not positive definite.

4. Show that if A is positive definite, so is A^{-1}. Hint: If $Ay = x$, $x^* A^{-1} x = y^* A^* y$.

5. Find the square root of the matrix:
$$A = \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}$$

6. Show that every rank one non-negative self-adjoint matrix can be written as
$$M = xx^*$$
where x is a complex column vector.

7. Construct two real positive 2×2 matrices, A, B whose symmetrized product, $S = AB + BA$ is not positive. Hint: see page 120.

8. Let $A \in M_n(C)$ be a positive semidefinite matrix and $x \in C^n$. Show that $x^* Ax = 0$ if and only if $Ax = 0$. Conclude that a positive semidefinite matrix A has rank n if and only if it is positive definite. Hint: Consider the quadratic polynomial $p(t) = (x + ty)^* A(x + ty)$. If $x^* Ax = 0$ show $p(t) \geq 0$ for all t, $p(0) = 0$ and $p'(0) = 0$. Conclude that $y^* Ax = 0$ for all $y \in C^n$ and thus, $Ax = 0$.

9. If A is self-adjoint, show that e^A is positive definite.