1. Show that the eigenvalues of a triangular matrix (all zeros above or below the main diagonal) are the diagonal elements.

2. Show that if $\lambda$ is an eigenvalue of $AB$ it is also an eigenvalue of $BA$ where $A, B$ are square matrices or equal dimension.

3. Show that if $I - AB$ is invertible then so $I - BA$ and that 
   \[(I - BA)^{-1} = I + B(I - AB)^{-1} A\]

4. Find the characteristic polynomial for:
   \[M = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -c & -b & -a \end{pmatrix}\]
   
   Suppose that $r$ is a root of the characteristic polynomial. Show that $v = [1, r, r^2]^T$ is the corresponding eigenvector.

5. Use the previous exercise to find the eigenvectors and eigenvalues for
   \[M = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\]
   find a matrix, $P$ such that
   \[PMP^{-1}\]
   is a diagonal matrix.

6. Let
   \[M = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\]
   Show that there is no matrix $P$ such that $PMP^{-1}$ is diagonal.

7. Let $T : P_2 \rightarrow P_2$ be defined as
   \[T(a + bt) = (4b - a) + (a - b)t.\]
   Find the eigenvalues and eigenvectors for $T$. 

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