Assignment 4

1. Problems 1 (p15), 3 (p19), 4 (p26)

2. Let $T : R^2 \to R^2$ be defined by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Prove

$$T^2 - (a + d)T + (ad - bc)I = 0$$

Use this to express $T^4$ in terms of $T, I$.

3. $T : R^3 \to R^2$ given by

$$T(x_1, x_2, x_3) = (x_1 - 2x_2 + x_3, x_3 - 4x_2)$$

(a) Write the matrix for $T$ using the standard bases for $R^3, R^2$

(b) Find a basis for the nullspace of $T$

(c) What is $T'$

4. Fix $T \in L(X, X)$. Prove that the set of $S \in L(X, X)$ which commutes with $T$ is a vector space. Show that the dimension of this space is at least 2 if the dimension of $X$ is greater than 1. Consider $X = R^2$ and

$$T = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$$

What is the basis for the set of matrices commuting with $T$. Find a basis for the annihilator of this subspace.

5. Suppose that $A, B$ are complex matrices. Show that $AB - BA = I$ is impossible.

6. Suppose $S$ is a linear operator on $R^2$ and $S^2 = S$. Show that either $S = I, S = 0$ or there is a basis for $R^2$ such that in terms of that basis,

$$S = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$