Homework 6. Due Feb 20

1. Consider the following model proposed many years ago by Gierer & Meinhardt:

\[ x' = r + \frac{x^2}{y} - x \]
\[ y' = -ay + x^2 \]

Does increase of \( x \) help (activate) or harm (inhibit) \( y \)? Does increase of \( y \) activate or inhibit \( x \)? This is called an activator-inhibitor system. Which is the activator and which is the inhibitor? \( r, a \) are positive constants. Find the equilibrium. Write down the Jacobian matrix and evaluate it at the equilibria. Find the determinant and trace. Let \( a = 0.25 \). For what values of \( r \) is the equilibrium stable. For what value of \( r \) is the trace 0 and thus there is a Hopf bifurcation? Numerically solve the equations for \( a = 0.25, r = 0.3, 0.2, 0.1 \) with \( x(0) = 0.13, y(0) = 0.45 \). For which of these is the equilibrium stable? I have made Matlab and XPP (my free software) code available. You can use XPP on the web from this link http://xpp.baumgartner.io/login, but you have to create an account. Here is another online one


2. Consider the following “snow shovel game.” Two drivers are driving down the road, when they encounter an impassable snow bank. They need to either wait until the city plow comes through or they can go ahead and shovel the snow themselves and get home earlier. The strategies are to shovel (S) or don’t shovel (D). The cost of shoveling is \( C \) which would be shared if both shovel. If only one person shovels, there is also a cost of waiting, \( W \) as it takes more time. Finally if neither shovels, then there is cost of waiting even longer, \( K > W \) since you have to wait for the city to come and plow. Write down the payoff matrix for this game. Based on the payoff matrix, find conditions on these costs such that (a) both shoveling is the Nash equilibrium (stable equilibrium for the replicator dynamics), or, (b) neither shoveling is a Nash equilibrium, or, (c) there is no pure strategy, and only a mixed strategy. You can do this easiest with the replicator dynamics.