Homework 4

1. Three's a crowd. Consider the following system of interactions:

\[ X \rightarrow * \]
\[ 2X \rightarrow 3X \]
\[ 3X \rightarrow 2X \]

at rates \( k_1, k_2, k_3 \) respectively. Use the laws of mass action to write the differential equations for \( X \). Show that \( X = 0 \) is always stable. Find conditions on the rate constants, \( k_{1,2,3} \) so that there are other positive roots and determine their stability. Graph the right-hand side for \( k_1 = 1, k_2 = 3, k_3 = 1 \) over \( 0 \leq x \leq 3 \) and graphically determine the stability of the equilibria.

2. Here is a simple linear negative feedback system:

\[ x' = -x - by \]
\[ y' = x - y \]

Show that \( b > 0 \) implies the origin is always stable. Now consider the 3 stage system:

\[ x' = -x - bz \]
\[ y' = x - y \]
\[ z' = y - z \]

(Think of it as if \( x \) helps \( y \), \( y \) helps \( z \) and \( z \) attacks \( x \).) Write the matrix for this and then write down the characteristic polynomial. Apply the Routh-Hurwitz criteria to see if \( 0 \) is stable for all \( b \) positive. You will find that if \( b \) is big enough, there is an oscillatory exponential growth. This is the basis of many famous feedback oscillators.