1. Smaller mammals and birds have faster heart rates than larger ones. Try to derive a scaling law that relates the heart rate to the animals size. Hint: assume the following (a) Energy lost is through heat loss on the body surface (b) energy gained is proportional to the blood flowing through the lungs which provides the oxygen (c) energy lost = energy gained. Let \( m \) be mass, \( r \) be pulse rate, \( S \) be surface area, and \( H \) be volume of blood pumped in one stroke. Make any assumptions about geometric similarity that you need to get something along the lines of \( r \propto m^d \). By taking logs of the data in the attached image, use least squares to approximation to fit the exponent. (This can be done quite easily using Excel or another spreadsheet program. In Excel, click on Tools Excel add ons and get the Analysis Tool Pack and the Solver. Type in the data. Then in separate columns, take the log of the data and then in use Tools Analysis to perform a regression. You could also plot the log data and add a trend line.)

2. In Gulliver’s Travels the Lilliputians decided to feed Gulliver 1728 times as much food as they ate. They reasoned that since Gulliver was 12 times their height, his volume was \( 12^3 = 1728 \) times theirs so he required 1728 times as much food. What is wrong with this reasoning? What is the correct answer? (see previous question).

3. A mouse dropped from a second story window will walk away without injury. A cat will break a few bones. A cow will turn to hamburger. Atmospheric drag is proportional to \( S v^2 \) where \( S \) is surface area and \( v \) is velocity. Kinetic energy is proportional to \( mv^2 \) where \( m \) is mass. Terminal velocity is that velocity where the drag force balances the force due to gravity (\( mg \)). (a) Show that the terminal velocity, \( v \) scales like \( m^{1/6} \) for similarly shaped objects. (b) Show that the kinetic energy per unit area scales as \( m^{2/3} \). (c) Use this to comment on what happens to small vs large animals if they fall down a great height. (d) Show that the height at which terminal velocity is reached scales like \( m^{1/3} \) (Hint recall from HS physics that potential energy is \( mgh \).)

4. The Froude number is a dimensionless number that is defined by the ratio of centripetal force to gravitational force, \( F = (mv^2/l) / (mg) \). Walking by animals is often modeled as an inverted pendulum. Empirically, it has been found that animals switch from a walk to a trot when \( F \) exceeds about 0.6. Use this to estimate the maximal walking speed of a person who has, a 1 meter long leg. I have a corgi with legs that are about 20 cm. At what speed will she switch to a trot? Use \( g = 9.8 m/sec^2 \). Note that 1 meter/second is 2.23 miles/hour.

5. Kleiber’s “law”, states that resting basal metabolism of an animal is empirically found to be \( b_b \propto m^{3/4} \) where \( m \) is the mass of the body. If we assume (as in problem 1) that energy is lost through the surface of the
body, then we should have that \( b = K m^{2/3} \) using geometric scaling. This says that energy use is completely inefficient and radiated out the body. The other extreme is that the energy use is 100% efficient and converted to work, in which case \( b = K' m \). Ballesteros et al (Scientific Reports, 2018,8:1448) suggest a double power law:

\[
b_C \propto f m^{2/3} + (1 - f)m
\]

where \( f \in [0,1] \) is the fraction of these two models. On a log-log plot ranging from \( m = 0.001 \) to \( m = 100 \) (a good range in Kg of mammals). Plot \( \log(b_c) \) and \( \log(b_K) \) for various values of \( f \), including the extremes, of \( f = 0, 1 \). Which value of \( f \) seems to best fit the relationship. (You can actually compute a best least square fit numerically, but it is not so easy.)

6. The cardiac systems of mammals have some nice scaling properties. The external work of the heart, \( E_W \) is dependent on the blood pressure and the stroke volume and has dimensions of Joules. The metabolic turnover rate, \( E_{MTR} \) has dimensions of joules/sec/kg. The mass of an animal is \( M \) (kg) and the heart rate is \( f_h \) (usually in 1/min). Use the Buckingham Pi theorem to derive a relationship between these 4 quantities. Recall that work (measured in joules, \( J \)) has the dimensions of force times distance. Once you have done this, apply your dimensionless ratios to the data for the three mammals: Man (\( M = 70 \) Kg, \( E_W = 1 \) joule, \( E_{MTR} = 1.17J/sec/kg, f_h = 70/min \)), Dog (\( M = 20 \) Kg, \( E_W = 0.44 \) joule, \( E_{MTR} = 1.6J/sec/kg, f_h = 90/min \)), and Guinea pig (\( M = 0.7 \) Kg, \( E_W = 0.0097 \) joule, \( E_{MTR} = 3.7J/sec/kg, f_h = 240/min \)). Caution: you will have to convert minutes to seconds!