1. A patient starts taking a drug once a day at an amount $b$ milligrams. From pharmacological studies, it is known that the drug's half-life is $h$ hours. (That is, the amount of drug decays exponentially such that after $h$ hours, it is half its original amount.) Let $a_n$ be the amount of drug right after the $n^{th}$ dose. Show that this satisfies:

$$a_{n+1} = ka_n + b$$

Figure out what $k$ should be based on the pharmacological data. Solve the difference equation assuming that $a_0 = 0$ by guessing that $a_n = c + d\lambda^n$. What is the steady state (equilibrium) value of the drug, that is the value as $n \to \infty$?

2. Fireflies from Southeast Asia (and also in the Great Smokey Mtns and in Allegheny National Forest) are known to congregate in large groups and flash synchronously. For the species, *Pteroptyx malaccae*, experimentalists have studied their behavior in the field and have estimated how the timing a single flash of light can shift the timing of their own flash. When left on his own (the animals that flash synchronously are males), the firefly will flash with a period of $P$ milliseconds. Let $T$ be the period of the strobe light that is being flashed. The shift in the fireflies time to flash is a sinusoidal function of the time that the stimulus arrives. So, let $\phi_n$ now be the phase (normalized time modulo the period, so that it lies between 0 and 1) that the animal flashes right before the $n^{th}$ stimulus. The above assumptions mean that:

$$\phi_{n+1} = \phi_n + \frac{T}{P} - \frac{a}{2\pi} \sin(2\pi\phi)$$

We define 1:1 locking to mean that the phase advances by exactly 1 between two flashes. That is $\phi_{n+1} = \phi_n + 1$. (a) Find conditions on $a$ and $T$ (the strobe period) such that there will be 1:1 locking and find the values $\phi$ such that locking occurs when it can occur (b) determine the stability of the locked solutions; (c) Pick $a = .5$ and $T/P = \{.96, 1.2, 1.9\}$ describe the behavior.

3. Birth death and thresholds. Suppose that a species needs enough of its own around or it will go extinct, and it will stop growing if it gets too crowded. This suggests the following model:

$$x_{n+1} = x_n + bx_n(x_n - r)(1 - x_n), \quad 0 < r < 1$$

(a) Give an interpretation of the parameters and terms in this model, (b) Find all the equilibria and their stability. (c) Sketch the graphical (cobweb) solution for $r = 0.25$ and $b = 0.5, 1.5, 2.5, 3.5$ and several different initial conditions for $x$. Are these results consistent with your stability analysis?
4. Consider the model for competition between two species:

\[ A_{n+1} = \mu_1 A_n - \mu_3 A_n B_n \]
\[ B_{n+1} = \mu_2 A_n - \mu_4 A_n B_n \]

where \( \mu_j \) are all positive constants. (a) Find all the non-negative equilibria.
(b) Determine their stability for the specific case, \( \mu_1 = 1.2, \mu_2 = 1.3, \mu_3 = 0.001, \mu_4 = 0.002 \).

5. A model for a host \((H)\)/parasite \((P)\) system takes the following form:

\[ H_{n+1} = H_n e^{1-H_n/K} e^{-aP_n} \]
\[ P_{n+1} = cH_n P_n \]

(a) Give an interpretation of the terms in the model. For example, first consider the host equation without the parasite \((P = 0)\). What is its behavior? What does the parasite do to the growth of the host? What is the effect of the host on the growth of the parasite? (b) Let \( h_n = H_n/K \) and \( p_n = aP_n \) be dimensionless variables. Show that the equations are then:

\[ h_{n+1} = h_n e^{1-h_n} e^{-p_n} \]
\[ p_{n+1} = dh_n p_n \]

and determine the value of \( d \) in terms of \( c, K, a \). (c) Find all the equilibria and determine their stability. (Hint, note that \( 1 = e^{1-h} e^{-p} \) can be easily solved by taking logs of both sides!). (d) Simulate the model for 200 iterations for with \( h_0 = 1, p_0 = 0.95 \) \( d = 0.9, d = 1.9, d = 2.2 \). Are your results compatible with your stability analysis? For example, for \( d = 2.2 \) are there any stable equilibria? If not how did they lose stability in the stability triangle?

Please note that there are a number of applets out there for cobwebbing.

https://mathinsight.org/applet/function_iteration_cobweb_combined
https://www.geogebra.org/m/QJ79IWCL
http://math.colgate.edu/math312/Spring1999/iterate.html