HW 9

1. Find all the Nash equilibria for the following payoff matrix

\[
\begin{pmatrix}
(5, 5) & (5, 10) & (8, 6) \\
(6, 8) & (4, 4) & (1, 3) \\
(3, 1) & (3, 1) & (7, 7)
\end{pmatrix}
\]

2. Consider the asymmetric rock, paper, scissors game with the following matrix (where \(a, b\) positive):

\[
\begin{pmatrix}
0 & -a & b \\
b & 0 & -a \\
-a & b & 0
\end{pmatrix}
\]

Let \(x, y, z\) denote the fraction of players using rock, paper, scissors respectively. Since \(z = 1 - x - y\), I want you to explore the dynamics of this for different choices of \(a, b\) in the \(x, y\) phaseplane using the replicator dynamics described in class. In particular, I want you to find all the equilibria and assess their stability (numerically, by probing with initial conditions near the equilibria). Consider the following choices of \((a, b) = (1, 1), (5, 1), (2, 1)\). I have included code for this in matlab and xpp (rps) (note that you will get the wrong answer in matlab for \(a = b = 1\) since it uses Euler and this is not accurate enough)

3. Make a table for the following scenario; an extension of Hawk-Dove. Let’s call it a Mixed strategy. In this case the player picks Hawk with probability \(p\) and dove with probability \(1 - p\). So we have \(H, D, M\). Consider \(H\) versus \(M\). When \(M\) plays \(H\), the payoff for both is \((G - C)/2\) and when \(M\) plays \(D\), the payoff for \(H\) is \(G\) and for \(M\) is 0. So this means that when \(H\) plays \(M\), the average payoff for \(H\) is \(p(G - C)/2 + G(1 - p)\). The payoff for \(M\) is \(p(G - C)/2 + (1 - p)0\). Based on this idea, fill in the rest of the table or payoffs. When \(D\) plays \(M\), it is also easy. The hard case is when \(M\) plays \(M\). There are 4 different scenarios, \((H, H), (H, D), (D, H), (D, D)\) with probabilities, \(p^2, p(1 - p), (1 - p)p, (1 - p)^2\) respectively. So this will yield the payoff for \(M\) playing \(M\). Now, suppose that \(G = 1\) and \(C = 2\). As we saw in class, for just \(H, D\), there is no pure strategy that is a Nash equilibrium. Can you find a \(p\) so that the \(M\) strategy is the Nash equilibrium? Note that this strategy will not be a strict Nash equilibrium. Instead, it is a strategy in which the payoff does not increase if you switch; it stays the same. More generally, show that for \(C > G\), the value of the probability is \(p = G/C\).