The tent map is defined as:

\[ f(x) = \begin{cases} \frac{m}{2} x & \text{for } 0 \leq x \leq \frac{1}{2} \\ m(1-x) & \text{for } \frac{1}{2} \leq x \leq 1 \end{cases} \]

a) Sketch the graph of \( f(x) \) for \( m > 0 \)
b) Find equilibria and their stability

c) Plot \( f \) for \( m = 2 \). Cobweb for \( m = 2 \) value
Ventilation problem.

Let $V_n$ be the volume at the $n$th breath and $C_n$ be the CO$_2$ in the body's blood. $\Delta$ CO$_2$ production $\rightarrow$ Amount Lost

$C_{n+1} = C_n + M - \Delta (V_n, C_n)$

$V_{n+1} = S(C_n) \leftrightarrow$ sensitivity to CO$_2$

Simple model: $L = \beta V_n \quad J = \lambda C_n$

$C_{n+1} = C_n + M - \beta V_n$

$V_{n+1} = \alpha C_n \Rightarrow V_n = \alpha C_{n-1}$

$C_{n+1} = C_n + M - \beta \alpha C_{n-1}$

How do we solve this? Just line does particular and homogeneous.

Look for $C_n = \overline{C}$ constant:

$\Rightarrow \overline{C} = \overline{C} + M - \alpha \beta \overline{C} \Rightarrow$

$\overline{C} = \frac{M}{\alpha \beta}$

Homogeneous problem:

$C_{n+1} = C_n - \alpha \beta C_{n-1}$

$\lambda^2 = \lambda - \alpha \beta \quad \lambda^2 - \lambda + \alpha \beta$

Roots are $\frac{1 \pm \sqrt{1 - 4\alpha \beta}}{2} = \rho^+, \rho^-$
\[ C_n = \frac{M}{\alpha \beta} + A (\rho^+)^n + B (\rho^-)^n \]

Assume \( 4\alpha \beta < 1 \) roots are:
\[
\frac{1}{2} + \sqrt{\frac{1-4\alpha \beta}{2}} , \quad \frac{1}{2} - \sqrt{\frac{1-4\alpha \beta}{2}}
\]

Both roots are positive and both are less than 1

\( (\rho^+)^n \to 0 \quad (\rho^-)^n \to 0 \)

\[ C_n \to \frac{M}{\alpha \beta} \]

If \( 4\alpha \beta > 1 \) then complex roots

If \( \frac{1}{4} < \alpha \beta < 1 \) complex damped

If \( \alpha \beta > 1 \) then complex undamped! growing
Nonlinear differential equation continued:

$$x_{n+1} = \frac{b \cdot x_n}{b + x_n} \quad b, k > 0$$

$$\bar{x} = \frac{k \cdot \bar{x}}{b + \bar{x}} \quad \Rightarrow \quad \bar{x} = 0 \text{ or } \bar{x} = k - b$$

**Stability**

$$f(x) = \frac{k \cdot x}{b + x} \quad f'(x) = \frac{k - k \cdot x}{(b + x)^2} = \frac{k - b}{(b + x)^2}$$

$$f'(0) = \frac{k}{b} \quad f'(k-b) = \frac{b}{k}$$

If $k < b$ then 0 is stable.
If $k > b$ then $k-b$ is stable.

Logistic equation revised:

$$x_{n+1} = a \cdot x_n \cdot (1-x_n)$$

$\bar{x} = 0$ stable if $0 < a < 1$

$\bar{x} = 1 - \frac{1}{a}$ stable if $1 < a < 3$

What happens for $a \geq 3$
When $a = 3 \quad \lambda = -1$

$y_{n+1} = -y_n \quad y_0 = c \quad c, -c, c, -c$

We say period 2 if $x_{n+2} = x_n$

for $\lambda$ slightly larger than $\lambda = 3 \quad \lambda < -1$

Nonlinearities keep it from blowing up.

- Period doubling

For example, if $a = 3.7 \quad \text{period 2}$

$x_n \rightarrow .799, .513, .799, .513 \ldots$

This period until $a \approx 3.449$

Then get period 4 point:

$x_{n+4} = x_n$

Period 4 until $a \approx 3.564$ period

Then period 8! etc

\[ 1 \quad 2 \quad 4 \quad 8 \quad 16 \ldots 
\]

\[ \text{chaos!!} \]
System of nonlinear differential equations

\( M = 2 \text{ dimension} \):

\[
\begin{align*}
x_{n+1} &= f(x_n, y_n) \\
y_{n+1} &= g(x_n, y_n)
\end{align*}
\]

Steady states:

\[
\begin{align*}
x &= f(x, y) \\
y &= g(x, y)
\end{align*}
\]

Linearization:

\[
A = \begin{bmatrix}
\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\
\frac{\partial g}{\partial x} & \frac{\partial g}{\partial y}
\end{bmatrix}
\]

\[
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\]

Eigenvalues \( \lambda^2 - (a_{11} + a_{22}) \lambda + a_{11}a_{22} - a_{12}a_{21} \)

\[
\text{Trace} = a_{11} + a_{22} \\
\text{Determinant} = a_{11}a_{22} - a_{12}a_{21}
\]

Refer to stability triangle!

\[
2 > 1 + \text{det} > |\text{Tr}|
\]

Example:

\[
\begin{align*}
x_{n+1} &= \alpha x_n + \beta x_n y_n \\
y_{n+1} &= \gamma y_n + \delta x_n y_n
\end{align*}
\]

Assume \( 0 < \delta < 1 \), \( \alpha > 1 \), \( \beta, \delta > 0 \)
Equilibrium:
\[ \bar{x} = \bar{x} (\alpha - \beta \bar{y}) \]
\[ \bar{y} = \bar{y} (\gamma + \delta \bar{x}) \]

\[ \bar{x} = \bar{y} = 0 \quad \text{otherwise} \]

\[ 1 = \gamma + \delta \bar{x} \Rightarrow \bar{x} = \frac{1}{\gamma} > 0 \]
\[ 1 = \alpha - \beta \bar{y} \Rightarrow \bar{y} = \frac{\alpha - 1}{\beta} > 0 \]
\[ \alpha - \beta \bar{y} = 1 ! \]

Stability
\[ f(x, y) = \alpha x - \beta xy \]
\[ g(x, y) = \gamma y + \delta xy \]

\[ (0, 0) \begin{bmatrix} \alpha & 0 \\ 0 & \gamma \end{bmatrix} \] eigenvalue \( \alpha > 1 \)
\[ \gamma > 1 \quad \text{UNSTABLE} \]

\[ A = \begin{bmatrix} 1 & -\frac{\beta (1-\gamma)}{\delta} \\ \frac{\delta}{\beta} (\alpha-1) & 1 \end{bmatrix} \]

\[ \text{Tr}(A) = 2 \quad \det A = 1 + (1-\gamma)(\alpha-1) > 1 \]
So ALways UNStable !