I have never seen it solved in the general form.[1] Let's do it quickly—the answer is revealing.

Let r denote the running speed, and s be the swimming speed. (Our units will be meters and seconds.) Let $T(y)$ represent the time to get to the ball given that Elvis jumps into the water at D, which is y meters from C. Let z represent the entire distance from A to C. Since time = distance/speed, we have

$$T(y) = \frac{z - y}{r} + \frac{\sqrt{x^2 + y^2}}{s}. \tag{1}$$

We want to find the value of y that minimizes $T(y)$. Of course this happens where $T'(y) = 0$. Solving $T'(y) = 0$ for y, we get

$$y = \frac{x}{\sqrt{r/s + 1}\,\sqrt{r/s - 1}}, \tag{2}$$

where T is seen to have a minimum by using the second derivative test. Several things about the solution should be noticed. First, somewhat surprisingly, the optimal path does not depend on z, as long as z is larger than y. Second, if $r < s$, we get no solution. That makes sense; if $r < s$ then it is obviously optimal to jump into the lake and swim the entire distance. Third, note that for $r \gg s$, y is small, and for $r \approx s$, y is large, as one would reasonably expect. Finally, note that for fixed r and s, y is proportional to x.

Now, back to Elvis. I noticed when playing fetch with Elvis that he uses the third strategy of jumping into the lake at D. It also seemed that his y values were roughly proportional to the x values. Thus, I conjectured that Elvis was indeed choosing the optimal path, and decided to test it by calculating his values of r and s and then checking how closely his ratio of y to x coincided with the exact value provided by the mathematical model.

With a friend to help me, we clocked Elvis as he chased the ball a distance of 20 meters on the beach. We then timed him as he swam (pursuing me) a distance of 10 meters in the water. His times are given in Table 1.

Table 1. Running and swimming times

Running times (in seconds) for 20 meters	Swimming times (in seconds) for 10 meters
3.20	12.13
3.16	11.15
3.15	11.07
3.13	10.75
3.10	12.22

Since we wanted Elvis's greatest running speed, we averaged just the three fastest running times, giving $r = 6.40$ meters/second. Similarly, using the three fastest swim-

[1]Several of the standard problems found in calculus texts are much more interesting and illuminating if done in the general case. For example, if you find the longest board that can be taken around a corner from a hallway of width a to a hallway of width b, you will discover a beautiful answer of the form $(a^p + b^p)^{1/p}$.

ming times, $s = 0.910$ meters/second. Then from (2), we get the predicted relationship that

$$y = 0.144x. \tag{3}$$

To test this relationship, I took Elvis to Lake Michigan on a calm day when the waves were small. I fixed a measuring tape about 15 meters down the beach at C from where Elvis and I stood at A as I threw the ball. After throwing it, I raced after Elvis, plunging a screwdriver into the sand at the place where he entered the water at D. Then I quickly grabbed the free end of the tape measure and raced him to the ball. I was then able to get both the distance from the ball to the shore, x, and the distance y. If my throw did not land close to the line perpendicular to the shoreline and passing through C, I did not take measurements. I also omitted the couple of times when Elvis, in his haste and excitement, jumped immediately into the water and swam the entire distance. I figured that even an "A" student can have a bad day. We spent three hours getting 35 pieces of data. We stopped only when the waves grew. Elvis had no interest in stopping or slowing down. The data are in Table 2.

Table 2. Throw and fetch trials

x	y	x	y	x	y	x	y	x	y
10.5	2.0	17.0	2.1	4.7	0.9	10.9	2.2	15.3	2.3
7.2	1.0	15.6	3.9	11.6	2.2	11.2	1.3	11.8	2.2
10.3	1.8	6.6	1.0	11.5	1.8	15.0	3.8	7.5	1.4
11.7	1.5	14.0	2.6	9.2	1.7	14.5	1.9	11.5	2.1
12.2	2.3	13.4	1.5	13.5	1.8	6.0	0.9	12.7	2.3
19.2	4.2	6.5	1.0	14.2	1.9	14.5	2.0	6.6	0.8
11.4	1.3	11.8	2.4	14.2	2.5	12.5	1.5	15.3	3.3

The scatter plot of these results is given in Figure 2.

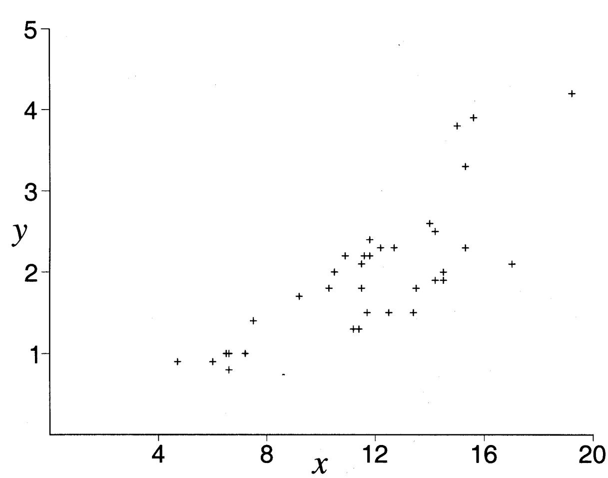

Figure 2. Scatter plot of Elvis's choices

Second, we confess that although he made good choices, Elvis does not know calculus. In fact, he has trouble differentiating even simple polynomials. More seriously, although he does not do the calculations, Elvis's behavior is an example of the uncanny way in which nature (or Nature) often finds optimal solutions. Consider how soap bubbles minimize surface area, for example. It is fascinating that this optimizing ability seems to extend even to animal behavior. (It could be a consequence of natural selection, which gives a slight but consequential advantage to those animals that exhibit better judgment.)

Finally, for those intrigued by this general study, there are further experiments that are available, other than using your own favorite dog. One might do a similar experiment with a dog running in deep snow versus a cleared sidewalk. Even more interesting, one might test to determine whether the optimal path is found by six-year-old children, junior high aged pupils, or college students. For the sake of their pride, it might be best not to include professors in the study.

Rational Boxes

Sidney Kung (shkung@tu.infi.net) shows how to find some nice integers. He writes:

I would like to suggest a simple approach to Philip K. Hotchkiss's Box Problem (this *Journal*, **33** (2002) #2, 113). My way of choosing positive integers a and b $(b > a)$ so that $c = \sqrt{a^2 - ab + b^2}$ is a positive integer is based on the following:

A formula similar to the one that generates Pythagorean triples,

$$(3m^2 - n^2)^2 + 3(2mn)^2 = (3m^2 + n^2)^2 \tag{1}$$

where m and n range over all positive integers, and if $a = p - q$ and $b = p + q$ for some positive integers p and $q (p > q)$, then

$$c = \sqrt{a^2 - ab + b^2} = \sqrt{q^2 + 3p^2}. \tag{2}$$

Since $n^2 - 3m^2 - 2mn = (n - 3m)(n + m)$, we choose $q = n^2 - 3m^2$ and $p = 2mn$ if $n > 3m$, so that $q > p$. On the other hand, since $3m^2 - n^2 - 2mn = (m - n)(3m + n)$, when $n < m$, we choose $q = 3m^2 - n^2$ and $p = 2mn$ so that $q > p$ holds also. Thus, from (1) we see that $c = \sqrt{a^2 - ab + b^2}$ is a positive integer.

For example, for $m = 1$ and $n = 4$ we get $q = 13$ and $p = 8$, so $(a, b, c) = (5, 21, 19)$. If $m = 5$ and $n = 3$, then $q = 66$, $p = 30$, and $(a, b, c) = (36, 96, 84)$, which is equivalent to $(3, 8, 7)$. My results seem to indicate that for any other triple either $a > 3$ or $a = 3$ and $b \geq 8$ would be true.