Are there bumps in cortex?

Re-evaluating the Mexican hat

Bard Ermentrout

University of Pittsburgh

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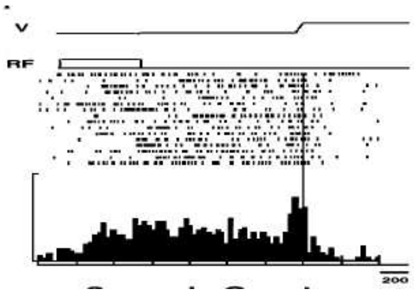
Persistent activity



- Delayed response tasks
- Occulomotor integrator
- Head direction system

The standard theory

- Persistent activity is an attractor
- Determined by recurrent connections
- Specific to the location - eg head angle, visual space
- **Spatially compact and stationary**

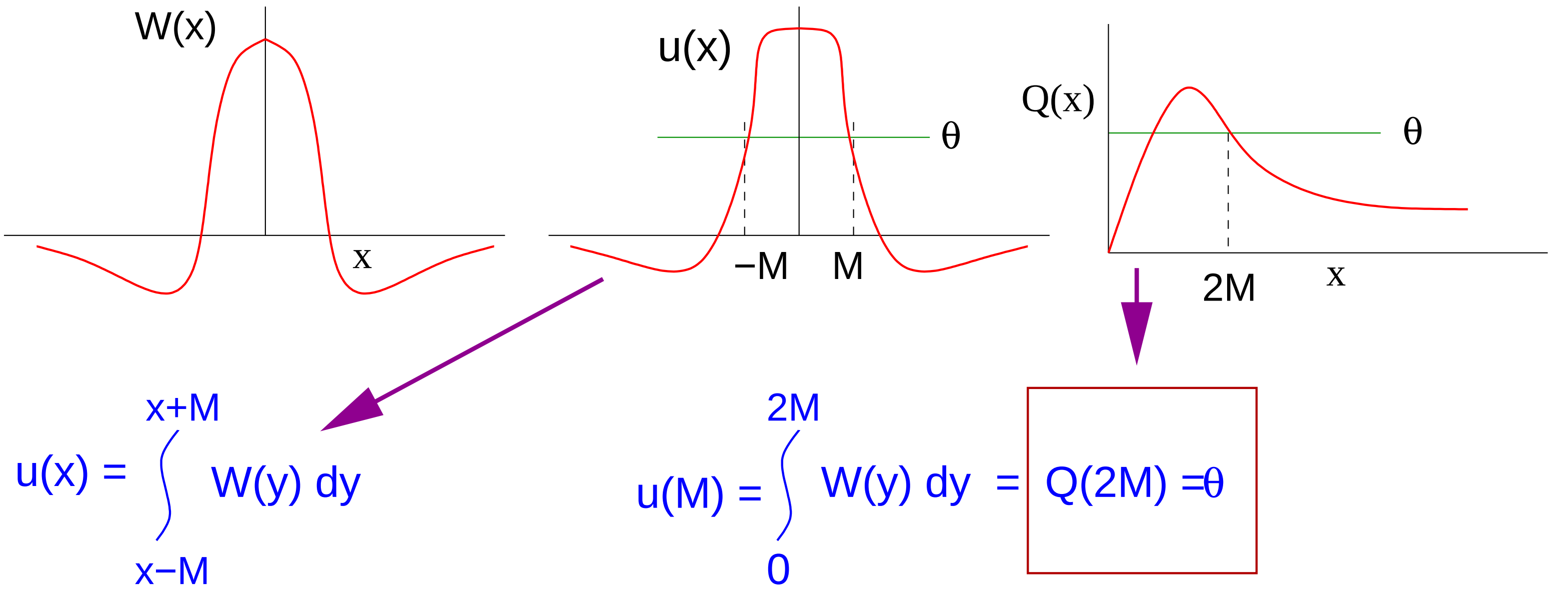
The standard model

- Strong recurrent excitation
- Stable rest state
- Lateral inhibition
- **Amari model**

$$u_t(x,t) = -u(x,t) + \int_{-\infty}^{\infty} W(x-y)F(u(y,t))\,dy$$

Amari's bumps

$$u(x) = \int_{-\infty}^{\infty} W(x-y) H(u(y) - \theta)\, dy$$



Physiology and anatomy

- **Lateral inhibition ?** Local excitatory connections extend farther than local inhibitory connections
- Why don't we see bumps in slices?
  - Plenty of persistent activity
  - Mostly waves

Possible counter-arguments

- Need some sort of modulators that aren't in slice
- Fast linear inhibition:

$$\begin{aligned} u(x) &= J_e(x) * F(u(x)) - a_{ie} J_i(x) * v(x) \\ v(x) &= a_{ei} J_e(x) * F(u(x)) \end{aligned}$$

implies

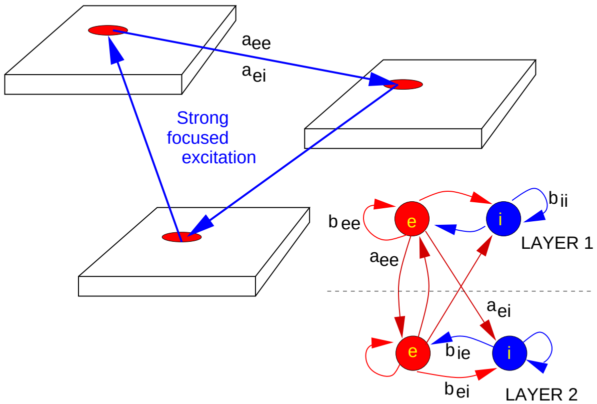
$$u(x) = [J_e(x) - a_{ie} a_{ei} J_e(x) * J_i(x)] * F(u(x))$$

which is a Mexican hat

- ... but inhibition is slower than excitation

Interactions between regions

- Cortex is organized in layers over many different areas
- Inputs from one area to the other are tightly focused
- Projections can be reciprocal with long feedback loops



Recurrent bistability

$$u'_j = -u_j + F_e(a_{ee}u_{j-1} + b_{ee}u_j - b_{ie}v_j)$$
$$\tau v'_j = -v_j + F_i(a_{ei}u_{j-1} + b_{ei}u_j - b_{ii}v_j)$$

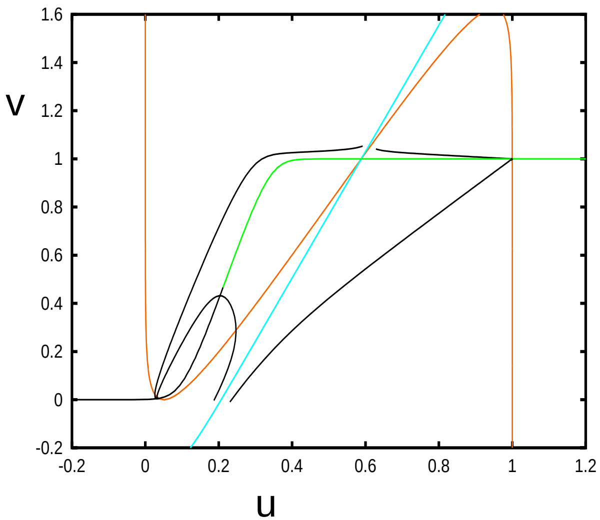
- Layer j receives **strong** excitatory projections from layer $j-1$ as well as its intrinsic local circuit connections.
- For example TC/RE <–> CTX
- If a_{ee} large enough then bistable

Symmetric local dynamics

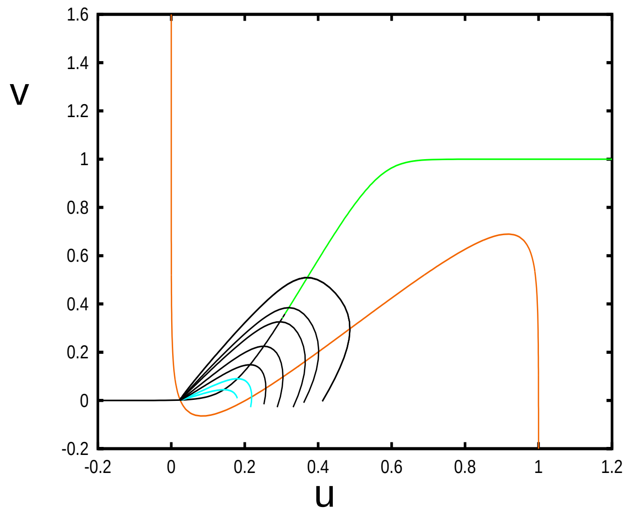
- Assume $u_j = u_k$ and $v_j = v_k$ to reduce to one pair of equations
-

$$u_t = -u + F_e((a_{ee}+b_{ee})u - b_{ei}v) \quad \tau v_t = -v + F_i((a_{ei}+$$

Normal



No layer−layer



Stability of symmetry

$$M = \begin{bmatrix} B_{ee} & A_{ee} & -B_{ie} & 0 \\ A_{ee} & B_{ee} & 0 & -B_{ie} \\ B_{ei} & A_{ei} & -B_{ii} & 0 \\ A_{ei} & B_{ei} & 0 & -B_{ii} \end{bmatrix}$$

decomposes into symmetric and asymmetric

$$M_S = \begin{bmatrix} B_{ee} + A_{ee} & -B_{ie} \\ B_{ei} + A_{ei} & -B_{ii} \end{bmatrix} \qquad M_A = \begin{bmatrix} B_{ee} - A_{ee} & -B_{ie} \\ B_{ei} - A_{ei} & -B_{ii} \end{bmatrix}$$

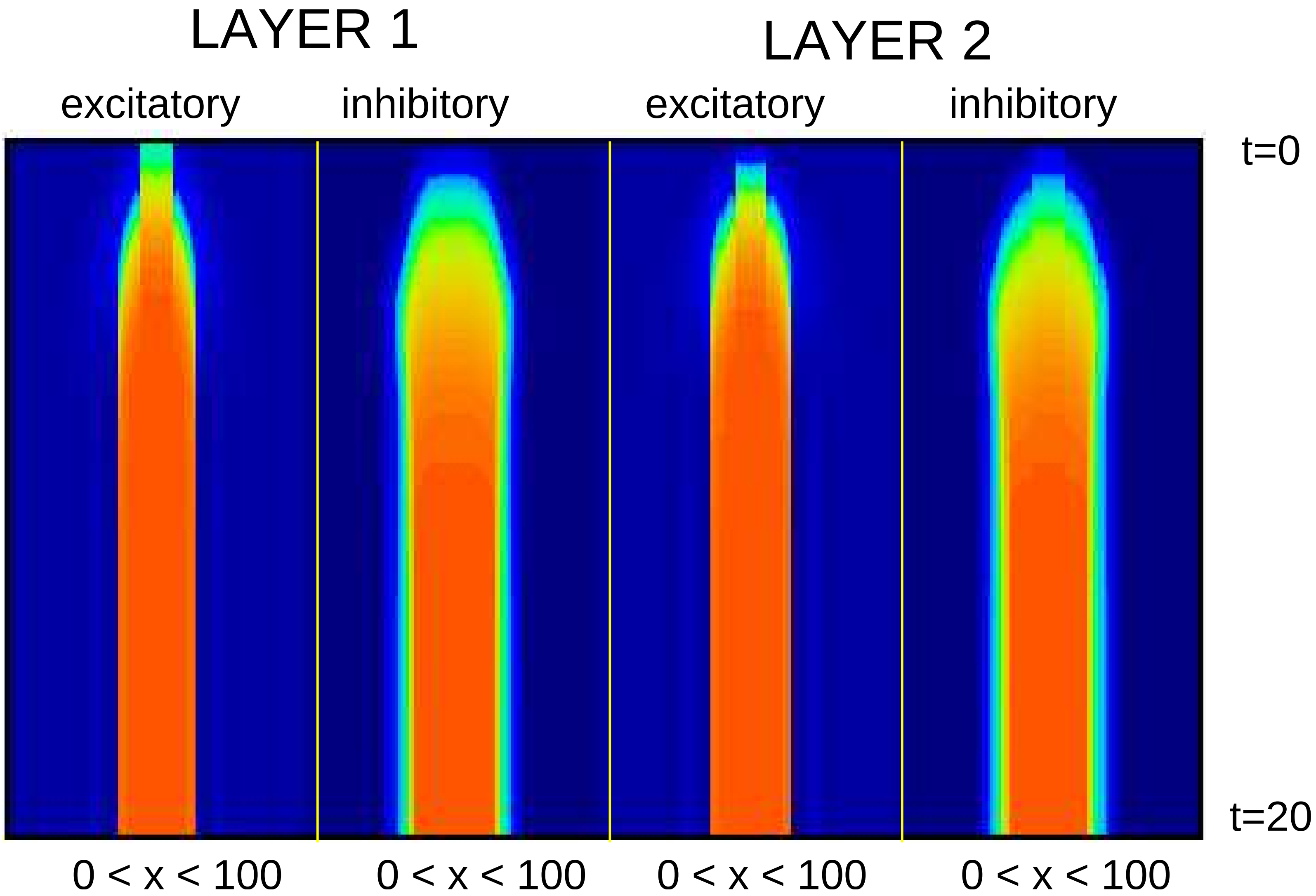
Need det$M_A > 0$ is det$M_S > 0$ which is true.

The full network

Here are the constraints:

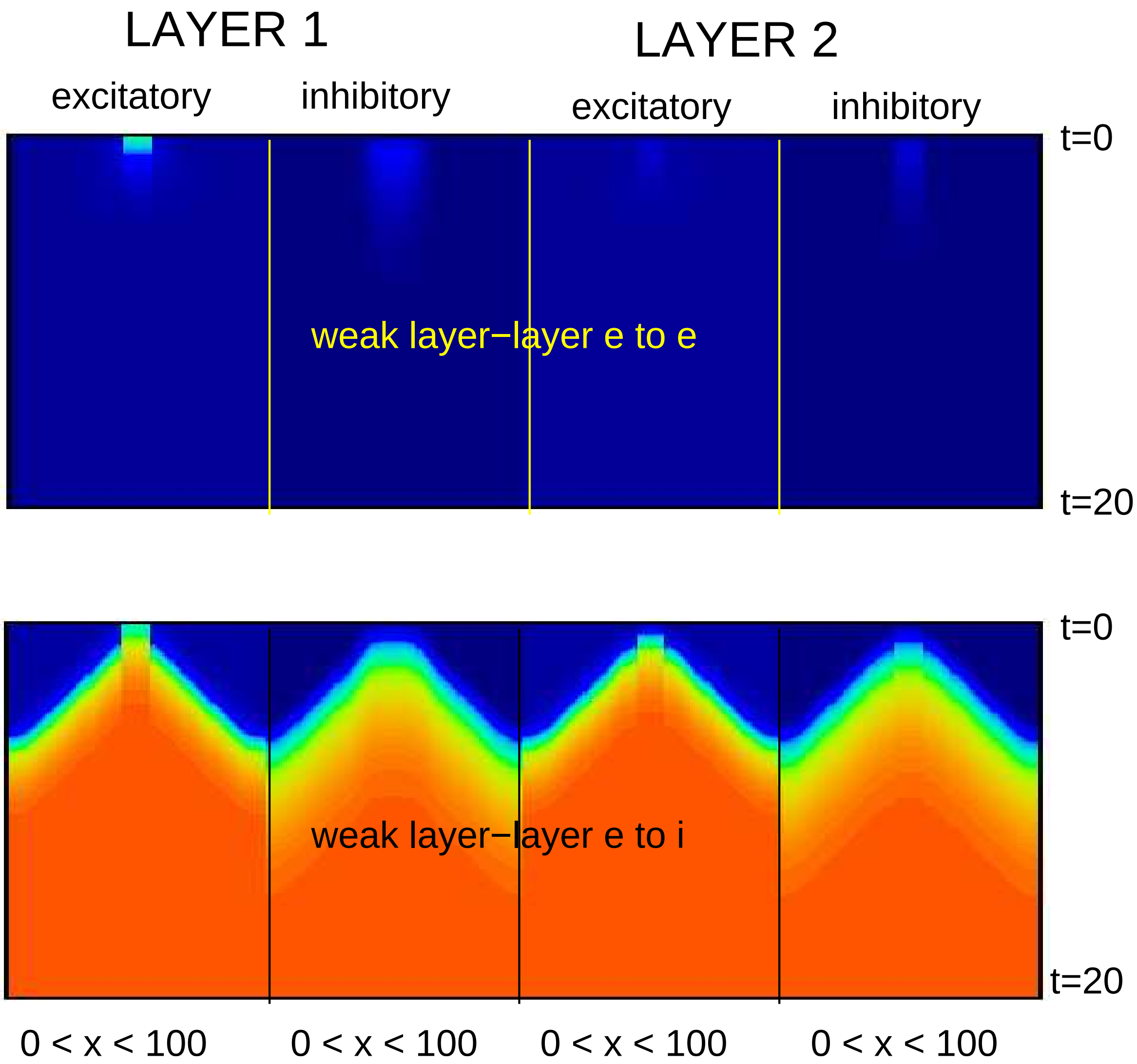
- Local inhibition spreads less than local excitation (anatomy)
- Bumps with strong layer-layer excitation (in vivo recordings)
- No bumps without layer-layer excitation (slice)
- Local waves with blocked inhibition (slice)

Bumps

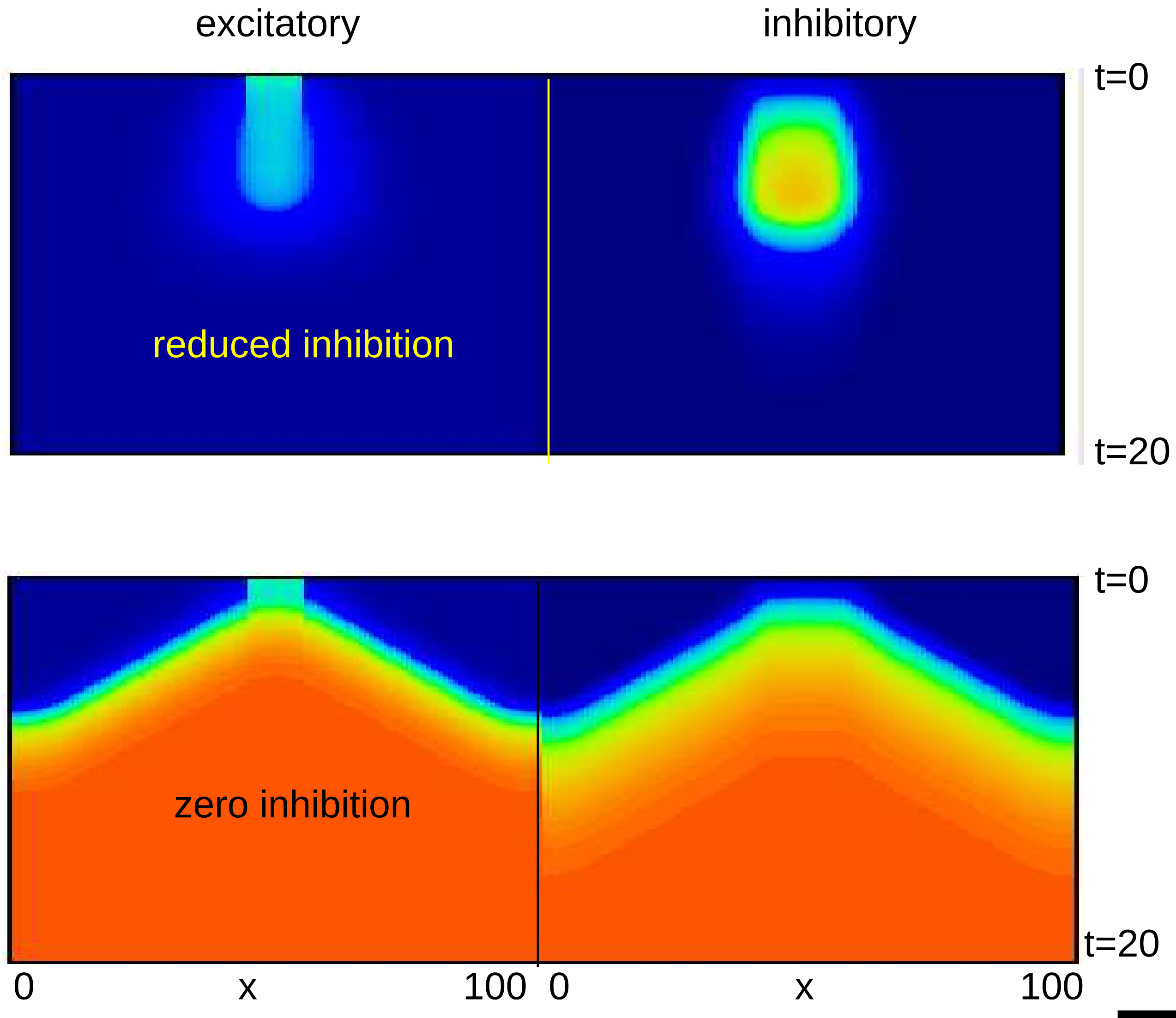


LAYER 1
LAYER 2
excitatory
inhibitory
excitatory
inhibitory
t=0
t=20
0 < x < 100
0 < x < 100
0 < x < 100
0 < x < 100

Manipulation of long-range connect



No long-range, local disinhibition



Analysis

- Assume weak local connections
- Steady state symmetric

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$$
\begin{aligned}
u(x) &= F_e(a_{ee}u(x)) + \epsilon F_e'(a_{ee}u(x)) \\
&\times [b_{ee}J_e(x) * u(x) - b_{ie}J_i(x) * v(x)] \\
v(x) &= F_i(a_{ei}u(x)) + \epsilon F_i'(a_{ei}v(x)) \\
&\times [b_{ei}J_e(x) * u(x) - b_{ii}J_i(x) * v(x)]
\end{aligned}
$$

Analysis

- Assume weak local connections
- Steady state symmetric

$$\begin{aligned} u(x) &= F_e(a_{ee}u(x)) + \epsilon F_e'(a_{ee}u(x)) \\ &\times [b_{ee}J_e(x) * u(x) - b_{ie}J_i(x) * v(x)] \\ v(x) &= F_i(a_{ei}u(x)) + \epsilon F_i'(a_{ei}v(x)) \\ &\times [b_{ei}J_e(x) * u(x) - b_{ii}J_i(x) * v(x)] \end{aligned}$$

- which simplifies to

Analysis cont'd

$$
\begin{aligned}
u(x) - F_e(a_{ee}u(x)) &= \epsilon F_e'(a_{ee}u(x)) \\
&\times \left[b_{ee}J_e(x) * u(x)\right. \\
&- \left. J_i(x) * b_{ie}F_i(a_{ei}u(x))\right]
\end{aligned}
$$

- Left-hand side is bistable between high and low values of $u(x)$
- Take $u(x) = u_{hi}$ in $0 < x < a$ and $u(x) = u_{lo}$ elsewhere
- For ϵ small enough this persists
- Note integrals smooth out the discontinuity
- Maybe use variant of Bates' theorem?