Low Mg^{++} Waves:
An Elliptic Burster?

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Biological motives

- Pinto induces waves with blocked inhibition
- What terminates activity?

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Biological motives

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  - Depolarization block may be one mechanism
Pinto induces waves with blocked inhibition

What terminates activity?
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In low Mg$^{++}$ wave is sometimes followed by slow oscillations

We suggest interaction between NMDAr & sodium inactivation
Expt’l depolarization block

- Intracellular recordings during wave
- Large synaptic current
- APs appear to be blocked
**Model**

- Golomb-Amitai single cell
- Type 1 spiking dynamics
- Has low threshold for depolarization block
- Used in past for waves in disinhibited slice
- Excitatory synapses only
The burster

\[ C \frac{dV}{dt} = -I_{ion} - g_s(V - E_{syn}) \]

\[ \frac{ds}{dt} = \left[ \alpha(V)(1 - s) - s \right]/\tau \]
The burster

\[ C \frac{dV}{dt} = -I_{ion} - g_s(V - E_{syn}) \]
\[ \frac{ds}{dt} = \frac{[\alpha(V)(1 - s) - s]}{\tau} \]

- Treat \( s \) as a slow variable
- Traverse upper branch
- Synaptic time course governs period
- Exists for small range of \( g_{syn} \) but period is fixed
Slow-fast picture

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\[ C \frac{dV_j}{dt} = -I_{ion}(V_j) - g_{syn} \left( \sum_k w_{j-k}s_k \right) [V - E_{syn}] \]

\[ \frac{ds_j}{dt} = \left[ \alpha(V_j)(1 - s_j) - s_j \right] / \tau \]
Simulation

Waves Splinters

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Splinters

- As time constant decreases, get a splintering of the wave
- Spatial patterns of localized activity
- “Negative coupling”
Splinters

- As time constant decreases, get a splintering of the wave
- Spatial patterns of localized activity
- "Negative coupling"

Normal form for coupled elliptic bursters

\[
\frac{\partial z}{\partial t} = z(c + a|z|^2 - |z|^4 - dK(x) * s(x))
\]

\[
\tau \frac{\partial s}{\partial t} = (|z|^2 - s)
\]
Simulation

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Explanation I.

- Let \( r = |z|^2 \) and look for stationary solns
- Get simple equation:

\[
0 = r(x)(c - dK(x) * r(x) + ar(x) - r(x)^2)
\]

- Solns include \( r(x) > 0 \) on \( x \in \Omega \) where

\[
\Omega = \bigcup_{\zeta}(\alpha_\zeta, \beta_\zeta)
\]

- “neutrally stable”
Explanation II.

Transition is best understood with a pair:

\[ r_j' = r_j(c - d(s_1 + s_2) + ar_j - r_j^2) + \epsilon \]
\[ s_j' = (r_j - s_j)/\tau \]

Bifurcation as $\tau$ decreases

Synchrony loses stability at pitchfork
Diagram

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Questions etc

- Can this be induced in real tissue
- If not, casts suspicion on depolarization block as only means of termination
- What determines splinter spacing?
Questions etc

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