Vectors

Vector $\vec{r} = <x, y>$ is a position vector with initial point the origin and tip at the point $(x,y)$.

Vectors $\vec{s}$ and $\vec{t}$ are equivalent if they have the same magnitude and direction, namely if $x = c - a = w - u$ and if $y = d - b = z - v$.

The sum of two vectors $\vec{a}$ and $\vec{b}$ geometrically is the diagonal of parallelogram formed by $\vec{a}$ and $\vec{b}$ with initial point the vertex of $\vec{a}$ and $\vec{b}$. It is calculated

$\vec{a} + \vec{b} = <a_1 + b_1, a_2 + b_2>$.

The difference of two vectors $\vec{a}$ and $\vec{b}$ geometrically is the diagonal of the parallelogram formed by vectors $\vec{a}$ and $\vec{b}$ with initial point the tip of $\vec{b}$ and extending to the tip of $\vec{a}$. It is calculated

$\vec{a} - \vec{b} = <a_1 - b_1, a_2 - b_2>$.

The vector $\vec{r} = <x, y>$ can be written in terms of the standard basis vectors $\hat{i}$ and $\hat{j}$ by $\vec{r} = x \hat{i} + y \hat{j}$.

A unit vector in the direction of $\vec{r}$ is determined by

$\frac{\vec{r}}{||\vec{r}||} = \frac{<x, y>}{\sqrt{x^2 + y^2}}$. 

The product of a vector \( \vec{a} = <a_1, a_2> \) and a scalar \( s \) is an \( s \) multiple of magnitude to the vector \( \vec{a} \) with the direction the same as \( \vec{a} \) if \( s \) is positive and opposite if \( s \) is negative.

\[ s \vec{a} = s <a_1, a_2> = <sa_1, sa_2> \]

The length (or magnitude) of a vector \( \vec{r} = <x, y> \) can be calculated by:

\[ \| \vec{r} \| = \sqrt{x^2 + y^2} \]

The direction of \( \vec{r} = <x, y> \) is the angle \( \theta \) determined by a counterclockwise rotation from the x-axis to the vector \( \vec{r} \).

The dot product of two vectors \( \vec{a} \) and \( \vec{b} \) is the scalar quantity:

\[ \vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 \]

The angle between the vectors \( \vec{a} \) and \( \vec{b} \) can be calculated by:

\[ \cos \theta = \frac{\vec{a} \cdot \vec{b}}{\| \vec{a} \| \| \vec{b} \|} \]

The scalar component of vector \( \vec{b} \) onto vector \( \vec{a} \) is the scalar quantity:

\[ \text{comp}_a \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\| \vec{a} \|} \| \vec{b} \| \cos \theta \]

The vector projection of vector \( \vec{b} \) onto vector \( \vec{a} \) is the vector quantity:

\[ \text{proj}_a \vec{b} = \left( \frac{\vec{a} \cdot \vec{b}}{\| \vec{a} \|^2} \right) \vec{a} \]
Given a two dimensional motion, \( \vec{r}(t) = <x(t), y(t)> \), the average velocity on the time interval \([t_1, t_2]\) is

\[
\vec{v}(t_1, t_2) = \frac{1}{t_2 - t_1} \left( \vec{r}(t_2) - \vec{r}(t_1) \right).
\]

The velocity vector, or tangent vector is

\[
\vec{v}(t) = \lim_{t_2 \to t} \frac{\vec{v}(t_2, t)}{t_2 - t_2} = \vec{r}'(t) = <x'(t), y'(t)>
\]

The speed of the object at time \( t \) is the magnitude of the velocity at time \( t \)

\[
\text{speed} = \| \vec{v}(t) \| = \| \vec{r}'(t) \|
\]

The parametric equation describing a line through a point with position vector \( \vec{r}_0 \) and parallel to the vector \( \vec{a} \) is

\[
\vec{r}(t) = \vec{r}_0 + s \vec{a} \quad s \in \mathbb{R}
\]

The parametric equation of the tangent line to the curve \( \vec{r}(t) = <x(t), y(t)> \) at \( t = t_1 \) is given by

\[
\vec{m}(s) = \vec{r}(t_1) + s \vec{r}'(t_1) \quad \text{or}
\]

\[
\vec{m}(s) = <x(t_1), y(t_1) > + s <x'(t_1), y'(t_1)>
\]

Uniform circular motion can be modeled by the equation

\[
\vec{r}(t) = r \cos \omega t, \sin \omega t \quad t \geq 0
\]

The velocity vector of motion is

\[
\vec{v}(t) = r \omega (-\sin \omega t, \cos \omega t)
\]

The speed of the object in motion is

\[
\text{speed} = \| \vec{r}'(t) \| = r \omega
\]

The acceleration vector of motion is

\[
\vec{a}(t) = \vec{r}''(t) = -r \omega^2 \vec{r}(t)
\]

The magnitude of the acceleration is

\[
a = \| \vec{r}''(t) \| = r \omega^2
\]

Let \( C \) be a smooth curve described by

\[
\vec{r}(t) = <x(t), y(t)>
\]

The arc length \( S \) of \( C \) is defined as

\[
s = \int_a^b \| \vec{r}'(t) \| \, dt = \int_a^b \sqrt{x'(t)^2 + y'(t)^2} \, dt
\]