Topic Review Exam II

1. Arclength

   (a) Polar Equations
   $$\text{Arclength}=\int_a^b \sqrt{(r(\theta))^2+(r'(\theta))^2}\,d\theta$$

   (b) Parametric Equations
   $$\text{Arclength}=\int_a^b \sqrt{(x'(t))^2+(y'(t))^2}\,dt$$

   (c) Functional $y=f(x)$
   $$\text{Arclength}=\int_a^b \sqrt{1+(f'(x))^2}\,dx$$

2. Unit Tangent Vectors

   (a) Parametric Equations: $\vec{r}(t)=<x(t),y(t)>$
   $$\vec{u}=\frac{1}{||\vec{r}\,'(t)||}\vec{r}\,'(t)$$

   (b) Polar Equations: $r(\theta)=g(\theta)$
   Converting to Parametric first gives $\vec{r}(t)=<g(t)\cos t, g(t)\sin t>$
   $$\vec{u}=\frac{1}{||\vec{r}\,'(t)||}\vec{r}\,'(t)$$

3. Area of Polar Regions
   $$\text{Area}=\int_a^b \frac{1}{2}(r(\theta))^2\,d\theta$$

4. Work

   (a) $$\text{Work}=\int_a^b \vec{F}\cdot d\vec{r}=\int_a^b \vec{F}\cdot\vec{r}\,'(t)\,dt$$

   (b) $$\text{Work}=\int_a^b \text{density}\times\text{volume}\times\text{distance}\,dy$$

   (c) $$\text{Work}=\int_a^b \text{Force}\times\text{distance}\,dx$$

5. Center of Mass $(\overline{x},\overline{y})$

$$\overline{x}=\frac{\int_a^b xf(x)\,dx}{\int_a^b f(x)\,dx} \qquad \overline{y}=\frac{\int_a^b \frac{1}{2}(f(x))^2\,dx}{\int_a^b f(x)\,dx}$$

6. Improper Integrals (convergence or divergence)
   (a) First Type: $\int_a^\infty f(x)\,dx$
   (b) Second Type: $\int_a^b \frac{f(x)}{g(x)}\,dx$ where $g(x) = 0$ on $a \le x \le b$
7. n^{th}-degree Taylor Polynomial
   $T_n(x, a) = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots + c_n(x-a)^n$
8. Geometric Series
   $\sum_{n=k}^{\infty} a \cdot r^n = \frac{a \cdot r^k}{1-r}$
9. Convergence
   (a) Ratio Test
   (b) Integral Test
   (c) Comparison Test
   (d) Alternating Series Test
10. Taylor Series of $f(x)$ about $x = a$
    $f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \cdots = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$
    so that $c_n = \frac{f^n(a)}{n!}$
11. Interval of Convergence
    Determine interval so that $\lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| < 1$
12. Radius of Convergence
    Half the length of the interval of convergence unless infinite