Review for Exam 2

1. Determine whether the given infinite series is convergent or divergent. In case of convergence, specify if it is absolute or conditional. Justify.

   (a) $\sum_{n=1}^{\infty} \frac{n+2}{n^2+3n-1}$ (b) $\sum_{n=1}^{\infty} \frac{\sqrt{t+1}}{t^2+3}$ (c) $\sum_{n!} 2^n(n+1)$

   (d) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+2}$ (e) $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n!}$ (f) $\sum_{n=1}^{\infty} \frac{2n-1}{n^3-4}$

2. Find the sums of the given convergent series.

   (a) $\sum_{n=0}^{\infty} \frac{9^{2n-1}}{4^{4n+2}}$ (b) $\sum_{n=0}^{\infty} \left(\frac{e}{\pi}\right)^n$ (c) $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n!}$

   (d) $\sum_{n=0}^{\infty} \frac{3^{n-1}}{2^{2n-1}}$ (e) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ (f) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

3. Find the interval of convergence of the given power series and find its sum, if possible, there.

   (a) $\sum_{n=0}^{\infty} \frac{(x+2)^n}{2^{n+1}}$ (b) $\sum_{n=0}^{\infty} \frac{x^{n+2}}{n!}$ (c) $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(n+1)!}$

4. Determine convergence or divergence of the given infinite series. Justify your answers.

   (a) $\sum_{n=1}^{\infty} \frac{1}{n^{1/n}}$ (b) $\sum_{n=1}^{\infty} \frac{n}{n+1}$ (c) $\sum_{n=1}^{\infty} \frac{n}{3^n}$ (d) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{x^2+3}}$

   (e) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+3}}$ (f) $\sum_{k=2}^{\infty} \left(\frac{\ln k}{k}\right)^k$ (g) $\sum_{n=1}^{\infty} \frac{n \cdot 3^n}{(n+1)!}$ (h) $\sum_{n=1}^{\infty} (-1)^n n$

5. Evaluate the improper integral if it converges. If it diverges, show reasons for divergence.

   (a) $\int_1^4 \frac{1}{(x-1)^2}\,dx$ (b) $\int_e^{\infty} \frac{1}{x\ln^2 x}\,dx$ (c) $\int_1^{\infty} \frac{\ln x}{x}\,dx$

   (d) $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$ (e) $\int_0^{\infty} \frac{x}{x^2+1}\,dx$ (f) $\int_0^{\infty} \frac{x}{(x^2+1)^2}\,dx$

6. Find the area A and the centroid $(\overline{x}, \overline{y})$ of the infinite lamina that is bounded by the curves $x = 0$, $y = 0$, and $y = e^{-x}$.

7. Consider the planar curve C, given by $\vec{r}(t) = \langle t - \sin t, 1 - \cos t\rangle$, $0 \le t \le \pi$. Find:

   (a) $\vec{v}$, $\vec{a}$, $||\vec{v}||$, $||\vec{a}||$ and κ for $t \in [0, \pi]$.

(b) T, and N, for $t \in [0, \pi]$

(c) T, and N, for $t = \pi/2$

(d) The arc-length L of C.

8. Find the arc-length L of the given curve C.

   (a) C is the cycloid $\vec{r}(t) = \langle \cos t + t \sin t, \sin t - t \cos t \rangle$, $0 \leq t \leq \pi$.

   (b) C is given by $y = x^{2/3}$, $0 \leq x \leq 8$.

   (c) C is given by $y = f(x)$, $-2 \leq x \leq 3$ where $f(x) = \begin{cases} x + 1, & x \leq 0 \\ x^{3/2} + 1, & x > 0 \end{cases}$

9. Find the MacLaurin expansion and its radius R of convergence for the given function $f(x)$.

   (a) $f(x) = x^2 e^x$ (b) $f(x) = \dfrac{x}{1-x}$

   (c) $f(x) = \dfrac{1}{1+x}$ (d) $f(x) = xe^{-x}$.

10. A right cylindrical tank of radius 4 ft and height 10 ft is half full of liquid with density ω lb/ft^3. Find the work W done by pumping the liquid to the top of the tank.

11. A spherical tank of radius 10 ft is full of liquid with density ω lb/ft^3. Find the work W done by pumping the liquid to the top of the tank.

12. Find the area A of the cardioid $r(\theta) = 1 - \sin \theta$.

13. Determine the area A of one petal of the three-petal rose, $r(\theta) = \sin(3\theta)$.

14. Find the area of the region R which lies in the first quadrant and is bounded by the polar curves $r(\theta) = \theta$ and $r(\theta) = \sin \theta$.

15. Find the work W done by stretching a spring 0.3m from its relaxed position, if the force exerted by the spring when it has been stretched 0.05 m has a magnitude of 25N.

16. An object moves from $(-1, 0)$ to $(1, 0)$ on the graph of $y = x^3 - x$, acted by a force that attracts it towards the point $(2, 0)$ and has the magnitude proportional to the distance between the object and the point $(2, 0)$. Calculate the work W done by the force, if at the point $(0, 0)$, the force has the magnitude of 6N.

17. Determine the sum S of the given series:

(a) $\sum_{n=0}^{\infty}\left(\frac{2}{9}\right)^n$ (b) $\sum_{n=0}^{\infty}\frac{2^{3n}}{3^{2n}}$ (c) $\sum_{n=0}^{\infty}\frac{2^n}{n!}$ (d) $\sum_{n=1}^{\infty}\frac{2^n}{n!}$

(e) $\sum_{n=0}^{\infty}\frac{3^{n+1}}{2^{2n+1}}$ (f) $\sum_{n=1}^{\infty}\left(\frac{2}{9}\right)^n$ (g) $\sum_{n=2}^{\infty}\left(\frac{2}{3}\right)^n$ (h) $\sum_{n=0}^{\infty}\frac{3^n}{n!}$

18. Tell whether the series is convergent or divergent. In case of convergence, specify if it is absolute or conditional. State the test of convergence/divergence that is used and justify your answer.

(a) $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ (b) $\sum_{n=1}^{\infty}\frac{(-1)^n}{5n-1}$ (c) $\sum_{n=1}^{\infty}(-1)^n\frac{n-2}{3n+1}$ (d) $\sum_{n=0}^{\infty}(-1)^n\frac{3^{3n}}{5^{2n}}$

(e) $\sum_{n=0}^{\infty}\frac{(-1)^n}{n+2}$ (f) $\sum_{n=1}^{\infty}\frac{n}{2^n}$ (g) $\sum_{n=1}^{\infty}(-1)^n\frac{n}{2^n}$ (h) $\sum_{n=1}^{\infty}\frac{n}{n+1}$

19. Use partial fractions to determine the sum S of the given series.

(a) $\sum_{n=1}^{\infty}\frac{1}{n^2+n}$ (b) $\sum_{n=0}^{\infty}\frac{1}{n^2+3n+2}$ (c) $\sum_{n=0}^{\infty}\frac{1}{n^2+4n+3}$

20. Determine the radius R an the interval of convergence for the given series.

(a) $\sum_{n=0}^{\infty}\left(\frac{2x}{3}\right)^n$ (b) $\sum_{n=1}^{\infty}\frac{(2x)^n}{3n}$ (c) $\sum_{n=1}^{\infty}\frac{(2x-1)^n}{n}$ (c) $\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}$

21. Use the geometric series to determine the power series for the given function $f(x)$. Also determine the corresponding radius and interval of convergence.

(a) $f(x)=\frac{1}{1-2x}$ (b) $f(x)=\ln(1-2x)$ (c) $f(x)=\frac{x}{1-2x}$

22. Use 22(b) to determine the power series and the interval of convergenec of the given function $f(x)$.

(a) $f(x)=\frac{x}{1+x^2}$ (b) $f(x)=\frac{x^2}{1+x^2}$ (c) $f(x)=\tan^{-1}(x)$

23. Find the Taylor expansion about $x=1$ and the corresponding interval of convergence of the given function $f(x)$.

(a) $f(x)=e^{2x}$ (b) $f(x)=\frac{1}{1+x}$ (c) $f(x)=\frac{x+1}{x+2}$