

1. Determine the Taylor Series of the function.

$$(a) f(x) = \sin(x) \text{ about } x = 0 \quad \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$(b) f(x) = \sin(x^2) \text{ about } x = 0 \quad \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{4n+2}$$

$$(c) f(x) = x^2 \sin(x^2) \text{ about } x = 0 \quad \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{4n+4}$$

$$(d) f(x) = \int x^2 \sin(x) dx \quad \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+4)(2n+1)!} x^{2n+4}$$

$$(e) f(x) = xe^{-x^3} \text{ about } x = 0 \quad \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{3n+1}$$

$$(f) f(x) = \int xe^{-x^3} dx \text{ about } x = 0 \quad \sum_{n=0}^{\infty} \frac{(-1)^n}{(3n+2)n!} x^{3n+2} + c$$

$$(g) f(x) = \ln(x) \text{ about } x = 1 \quad \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} (x-1)^{n+1}$$

$$(h) f(x) = \int \frac{\ln x}{x-1} dx \quad \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)^2} (x-1)^{n+1} + c$$

$$(i) f(x) = \sqrt{x} \text{ about } x = 4 \quad 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3$$

$$(j) f(x) = \cos(3x) \quad \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n}}{(2n)!} x^{2n}$$

$$(k) f(x) = x^2 \cos(3x) \quad \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n}}{(2n)!} x^{2n+2}$$

2. Determine a series to give the following evaluation. Write out the first four terms to approximate this evaluation..

$$(a) \int_0^1 x^2 \sin(x^2) dx$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+5)(2n+1)!} = \frac{1}{5} - \frac{1}{54} + \frac{1}{(13)(120)} - \frac{1}{(17)(7!)} + \dots$$

$$(b) \int_0^1 x e^{-x^3} dx$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(3n+2)n!} = \frac{1}{2} - \frac{1}{5} + \frac{1}{16} - \frac{1}{66} + \dots$$

$$(c) \int_0^1 e^{-x^2} dx$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)n!} = 1 - \frac{1}{3} + \frac{1}{10} - \frac{1}{42} + \dots$$

3. Determine the sum:

$$(a) \sum_{n=0}^{\infty} \frac{3^n}{n!} \quad \underline{= e^3}$$

$$(b) \sum_{n=0}^{\infty} -\frac{2^n}{3^n \cdot n!} \quad \underline{= -e^{2/3}}$$

$$(c) \sum_{n=0}^{\infty} \frac{3}{2^n \cdot n!} \quad \underline{= 3\sqrt{e}}$$

$$(d) \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \quad \underline{= \cos(1)}$$

$$(e) \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \quad \underline{= \sin(1)}$$