PRACTICE8 Name______________________________________ Section______

1. Determine a direct formula for S_n, the nth term in the sequence of partial sums. Then use this formula to determine the sum S of the series. Note $S = \lim_{n\to\infty} S_n$.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{4}{n^2+4n+3}$

$S_n = 2\left(1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right)$ $\quad S = 3.$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(9^{\frac{1}{n}} - 9^{\frac{1}{n+3}}\right)$

$S_n = 9 + 9^{1/2} + 9^{1/3} - 9^{1/(n+1)} - 9^{1/(n+2)} - 9^{1/(n+3)}$ $\quad S = 9 + \sqrt[3]{9}.$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n}}{5^{n-1}}$

$S_n = 20 - \frac{4^{n+1}}{5^{n-1}}$ $\quad S = 20.$

(d) $\frac{1}{2} - \frac{2}{5} + \frac{8}{25} - \frac{32}{125} + \frac{128}{625} - \frac{512}{3125} + \cdots$

$S_n = \frac{5}{9}\left(\frac{1}{2} - \frac{(-1)^n\, 4^n}{2\cdot 5^n}\right)$ $\quad S = \frac{5}{18}$

2. Determine the interval of convergence of the given series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{4^n\, x^n}{3^n}$ $\quad \underline{-\frac{3}{4} < x < \frac{3}{4}}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{4^n\, x^n}{n!}$ $\quad \underline{-\infty < x < \infty}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n}}{(2n)!}$ $\quad \underline{-\infty < x < \infty}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-3)^n}{(n+1)^2}$ $\quad \underline{2 \le x \le 4}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{(n+1)\, 3^n}$ $\quad \underline{-3 \le x < 3}$

3. Write the given function as a power series. Be sure to stat the interval for which this function is equal to its power series.

(a) $f(x) = \dfrac{4}{1+3x}$ $\quad \sum_{n=0}^{\infty} 4(-1)^n\,(3)^n\,x^n \quad -\frac{1}{3} < x < \frac{1}{3}$

(b) $f(x) = \dfrac{4}{(1+3x)^2}$ $\quad \sum_{n=1}^{\infty} 4(-1)^n\,(3)^{n-1}\,n\,x^{n-1} \quad -\frac{1}{3} < x < \frac{1}{3}$

(c) $f(x) = 4\ln(1+3x)$ $\quad \sum_{n=0}^{\infty} \dfrac{4(-1)^n\,(3)^{n+1}}{n+1}\,x^{n+1} \quad -\frac{1}{3} < x \le \frac{1}{3}$

(d) $f(x) = \dfrac{6}{x+8}$ $\quad \sum_{n=0}^{\infty} \dfrac{3(-1)^n}{4\cdot 8^n}\,x^n \quad -8 < x < 8$

(e) $f(x) = \dfrac{6}{(x+8)^2}$ $\quad \sum_{n=1}^{\infty} \dfrac{3(-1)^{n-1}\,n}{4\cdot 8^n}\,x^{n-1} \quad -8 < x < 8$

(f) $f(x) = \dfrac{6}{(x+8)^3}$ $\quad \sum_{n=2}^{\infty} \dfrac{3(-1)^n\,n\,(n-1)}{8^{n+1}}\,x^{n-2} \quad -8 < x < 8$

(g) $f(x) = \dfrac{5}{1+4x^2}$ $\quad \sum_{n=0}^{\infty} 5(-1)^n\,(4)^n\,x^{2n} \quad -\frac{1}{2} < x < \frac{1}{2}$

(h) $f(x) = 5\arctan(2x)$ $\quad \sum_{n=0}^{\infty} \dfrac{5(-1)^n\,(2)^{2n+1}}{2n+1}\,x^{2n+1} \quad -\frac{1}{2} \le x \le \frac{1}{2}$

(i) $f(x) = \dfrac{5x}{1+4x^2}$ $\quad \sum_{n=0}^{\infty} 5(-1)^n\,(4)^n\,x^{2n+1} \quad -\frac{1}{2} < x < \frac{1}{2}$

(j) $f(x) = 5\ln(1+4x^2)$ $\quad \sum_{n=0}^{\infty} \dfrac{40(-1)^n\,(4)^n}{2n+2}\,x^{2n+2} \quad -\frac{1}{2} \le x \le \frac{1}{2}$