PRACTICE7 Name____________________________________ Section______

1. Determine whether the sequence converges or diverges. If it converges, give the limit of the sequence.

   (a) $\left\{1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, \ldots\right\}$ converges and limit is $L = 0$

   (b) $\left\{\frac{3n}{4n+1}\right\}_{n=1}^{\infty}$ converges and limit is $L = \frac{3}{4}$

   (c) $\left\{\frac{\cos(n\pi)}{n}\right\}_{n=1}^{\infty}$ converges and limit is $L = 0$

   (d) $\left\{\frac{(-1)^n (n+1)}{n}\right\}_{n=1}^{\infty}$ diverges

   (e) $\left\{\frac{3^{n+1}}{4^{n-1}}\right\}_{n=1}^{\infty}$ converges and limit is $L = 0$

   (f) $\left\{\frac{3^{3n+1}}{4^{2n-1}}\right\}_{n=1}^{\infty}$ diverges

2. Determine the sum of the series:

   (a) $\sum_{n=0}^{\infty} 4\left(\frac{2}{5}\right)^n$ $\quad S = \frac{4}{1-(2/5)} = \frac{20}{3}$

   (b) $\sum_{n=0}^{\infty} 3\left(-\frac{7}{8}\right)^n$ $\quad S = \frac{3}{1-(-7/8)} = \frac{8}{5}$

   (c) $\sum_{n=0}^{\infty} \frac{2^{2n}}{5^{n+1}}$ $\quad S = \frac{1/5}{1-(4/5)} = 1$

   (d) $\sum_{n=0}^{\infty} \frac{3^{2n+2}}{4^{2n}}$ $\quad S = \frac{9}{1-(9/16)} = \frac{144}{7}$

   (e) $\sum_{n=1}^{\infty} \frac{4}{n^2+5n+4}$ $\quad S_n = \frac{4}{3}(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} - \frac{1}{n+2} - \frac{1}{n+3} - \frac{1}{n+4})$ $\quad S = \frac{13}{9}$

   (f) $\sum_{n=3}^{\infty} \frac{3}{n^2-2n}$ $\quad S_n = \frac{3}{2}(1 + \frac{1}{2} - \frac{1}{n-1} - \frac{1}{n})$ $\quad S = \frac{9}{4}$

   (g) $\sum_{n=1}^{\infty} \left(3^{1/n} - 3^{1/n+2}\right)$ $\quad S_n = 3^1 + 3^{1/2} - 3^{1/(n+1)} - 3^{1/(n+2)},$ $\quad S = 1 + \sqrt{3}$

3. Show whether the series converges or diverges by reason of a valid convergence test.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{4n}{1+9n^2}$ Diverges. The Integral/Comparison Test shows:

$$\int_0^{\infty} \frac{4x}{1+9x^2}\,dx = \frac{2}{9}\ln\left(1+9x^2\right)\Big|_0^{x=t\to\infty} \to \infty \text{ which diverges.}$$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{4}{1+9n^2}$ Converges. The Integral/Comparison Test shows:

$$\int_0^{\infty} \frac{4}{1+9x^2}\,dx = \frac{4}{3}\arctan(3x)\Big|_0^{x=t\to\infty} = \frac{2\pi}{3} \text{ which converges.}$$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{4\cos(2n)}{n^2}$ Converges. Comparing: $\displaystyle < \sum_{n=1}^{\infty} \frac{4|\cos(2n)|}{n^2} < \sum_{n=1}^{\infty} \frac{4}{n^2}$

$$\int_1^{\infty} \frac{4}{x^2}\,dx = -\frac{4}{x}\Big|_1^{x=t\to\infty} = 4 \text{ which converges.}$$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{n^3+n^2+1}$ Converges. Comparing: $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2}$

$$\int_1^{\infty} \frac{2}{x^2}\,dx = -\frac{2}{x}\Big|_1^{x=t\to\infty} = 2 \text{ which converges.}$$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\,7n}{1+9n^2}$ Converges. Alternating so by the Alternating Series Test:

$$b_{n+1} < b_n \text{ and } \lim_{n\to\infty} \frac{7n}{1+9n^2} = 0.$$

(f) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n\,7n}{1+9n}$ Diverges. Alternating so by the Alternating Series Test:

$$\lim_{n\to\infty} \frac{7n}{1+9n} = \frac{7}{9} \neq 0.$$

(g) $\displaystyle\sum_{n=0}^{\infty} \frac{n^3\cdot 2^n}{n!}$ Converges. The Ratio Test shows:

$$\lim_{n\to\infty} \left|\frac{(n+1)^3\cdot 2^{n+1}}{(n+1)!}\cdot\frac{n!}{n^3\cdot 2^n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^3\cdot 2}{n^3\cdot(n+1)}\right| = 0 < 1.$$

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{n\cdot 2^{n+2}}{3^{n+1}}$ Converges. The Ratio Test shows:

$$\lim_{n\to\infty} \left|\frac{(n+1)\cdot 2^{n+3}}{3^{n+2}}\cdot\frac{3^{n+1}}{n\cdot 2^{n+2}}\right| = \lim_{n\to\infty}\left|\frac{2(n+1)}{3n}\right| = \frac{2}{3} < 1.$$

(i) $\sum_{n=0}^{\infty} 6e^{-n}$ Converges. The Ratio Test shows:

$$\lim_{n\to\infty} \left| \frac{6e^{-(n+1)}}{6e^{-n}} \right| = \frac{1}{e} < 1.$$