

1. Determine the angle between the vectors: (you can use a calculator)

$$(a) \langle 4, 3 \rangle \text{ and } \langle -4, 2 \rangle \quad \underline{\cos^{-1}\left(\frac{-1}{\sqrt{5}}\right)}$$

$$(b) \langle -3, 2 \rangle \text{ and } \langle -8, -12 \rangle \quad \underline{\cos^{-1}\left(\frac{0}{\sqrt{13}\sqrt{208}}\right) = 90^\circ}$$

$$(c) \langle 5, 4 \rangle \text{ and } \langle 1, 9 \rangle \quad \underline{\cos^{-1}\left(\frac{41}{\sqrt{41}\sqrt{82}}\right) = 45^\circ}$$

$$(d) \langle 2, 1, 8 \rangle \text{ and } \langle 1, -3, 2 \rangle \quad \underline{\cos^{-1}\left(\frac{15}{\sqrt{69}\sqrt{14}}\right)}$$

2. Determine the unit vector in the direction of:

$$(a) \langle 5, -12 \rangle \quad \underline{\vec{u} = \left\langle \frac{5}{13}, -\frac{12}{13} \right\rangle}$$

$$(b) -3\vec{i} + 5\vec{j} + 8\vec{k} \quad \underline{\vec{u} = -\frac{3}{\sqrt{98}}\vec{i} + \frac{5}{\sqrt{98}}\vec{j} + \frac{8}{\sqrt{98}}\vec{k}}$$

3. Determine the scalar and vector projection of \vec{F} onto \vec{d} if:

$$\vec{F} = \langle 1, 8 \rangle \text{ and } \vec{d} = \langle 12, 5 \rangle \quad \underline{sp = \frac{42}{13}} \quad \underline{v\vec{p} = \frac{42}{169}\langle 12, 5 \rangle}$$

4. Determine the cross product of the vectors:

$$(a) \langle 3, 5, -1 \rangle \text{ and } \langle -2, -3, 4 \rangle$$

$$\begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 5 & -1 \\ -2 & -3 & 4 \end{pmatrix} = 17\vec{i} - 10\vec{j} + 1\vec{k}$$

$$(b) \langle 0, 5, 8 \rangle \text{ and } \langle 4, 0, -2 \rangle$$

$$\begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 5 & 8 \\ 4 & 0 & -2 \end{pmatrix} = -10\vec{i} + 32\vec{j} - 20\vec{k}$$

5. Determine the area of the parallelogram formed by the vectors $\vec{a} = \langle 5, 2 \rangle$ and $\vec{b} = \langle -1, 7 \rangle$.

$$\begin{vmatrix} 5 & 2 \\ -1 & 7 \end{vmatrix} = 37$$

6. Determine the area of the parallelogram formed by the vectors $\vec{a} = \langle 4, 5, 8 \rangle$ and $\vec{b} = \langle 1, -4, 5 \rangle$.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 5 & 8 \\ 1 & -4 & 5 \end{vmatrix} = |57\vec{i} - 12\vec{j} - 21\vec{k}| = \sqrt{57^2 + 12^2 + 21^2}$$

7. Determine the volume of the parallelepiped formed by the vectors $\vec{a} = \langle 3, -4, 1 \rangle$, $\vec{b} = \langle 5, 2, 4 \rangle$, and $\vec{c} = \langle -1, 3, 3 \rangle$.

$$\begin{vmatrix} 3 & -4 & 1 \\ 5 & 2 & 4 \\ -1 & 3 & 3 \end{vmatrix} = |3(-6) + 4(19) + 1(17)| = 75$$

8. Determine the equation of the plane determined by the vectors $\vec{v} = \langle 2, -1, 4 \rangle$ and $\vec{w} = \langle 6, 3, 1 \rangle$ and passes through the point $P(3, -2, 2)$.

$$\vec{n} = \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 4 \\ 6 & 3 & 1 \end{pmatrix} = -13\vec{i} + 22\vec{j} + 12\vec{k}$$

Plane: $-13(x - 3) + 22(y + 2) + 12(z - 2) = 0$

9. Determine the equation of the plane passing through the points $P(1, 2, -4)$, $Q(0, 3, 7)$ and $R(2, -2, 1)$.

$$\vec{PQ} = \langle -1, 1, 11 \rangle \text{ and } \vec{PR} = \langle 1, -4, 5 \rangle$$

$$\begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 11 \\ 1 & -4 & 5 \end{pmatrix} = 49\vec{i} + 16\vec{j} + 3\vec{k}$$

Plane: $49(x - 2) + 16(y + 2) + 3(z - 1) = 0$