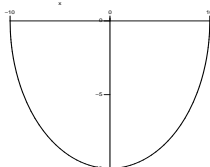


1. Determine the work to empty the water from a filled semi-spherical swimming pool with radius 10 ft.



$$D_y = -y \text{ ft}$$

$$F_y = 62.4\pi(\sqrt{100 - y^2})^2 \Delta y \text{ lbs}$$

$$\begin{aligned} W &= \int_{-10}^0 62.4\pi(y^3 - 100y) dy \\ &= 62.4\pi \left[ \frac{1}{4}y^4 - \frac{100}{2}y^2 \right]_{-10}^0 \\ &= 62.4\pi(2500) \text{ ft-lbs.} \end{aligned}$$

2. A cable that weighs 2 lb/ft is used to lift 800 lb of coal up a mine shaft 500 ft deep. Find the work done.

$$W_{\text{coal}} = 800(500) \text{ ft-lbs} = 400,000 \text{ ft-lb}$$

$$\begin{aligned} W_{\text{rope}} &= \int_0^{500} 2(500 - y) dy \\ &= 2 \left( 500y - \frac{1}{2}y^2 \right)_0^{500} \\ &= 250,000 \text{ ft-lbs} \end{aligned}$$

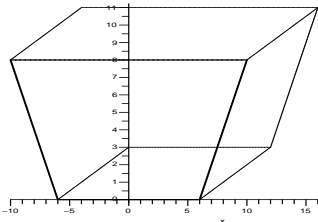
$$\text{Work} = 650,000 \text{ ft-lb}$$

3. A spring has natural length 20 cm. If a 25-N force is required to keep it stretched to a length of 30 cm (determining  $k$ , the spring constant), how much work is required to stretch it from 20 cm to 25 cm?

$$F = kx \Rightarrow 25 = k(.3) \Rightarrow k = 250/3$$

$$\begin{aligned} W &= \int_{1/5}^{1/4} \frac{250}{3}x dx \\ &= \frac{250}{3}x^2 \Big|_{1/5}^{1/4} \\ &= \frac{250}{3} \left( \frac{1}{16} - \frac{1}{25} \right) \text{ J} \end{aligned}$$

4. SET UP (do not evaluate) the integral to determine the force on the trapezoidal side of the tank filled with water that has a rectangular bottom with length 12 ft and width 10 ft and rectangular top with length 20 ft and width 10 ft. The tank is 8 ft deep from bottom to top.



Equation of the line having points  $(6, 0)$  and  $(10, 8)$  is  $y = 2x - 12$

$$A_y = (2) \left( \frac{y+2}{2} \right) \Delta y$$

$$P_y = 62.4(8 - y)$$

$$F = \int_0^8 62.4(8 - y)(y + 12) dy$$

5. SET UP (do not evaluate) the integral to determine the work to empty this trapezoidal tank.

$$F_y = 62.4(10)(2) \left( \frac{y+2}{2} \right) \Delta y$$

$$D_y = (8 - y)$$

$$W = \int_0^8 624(8 - y)(y + 12) dy$$

6. Determine the arclength of the curve  $f(x) = \frac{x^2}{2} - \frac{\ln x}{4}$  for  $1 \leq x \leq 4$ .

$$f'(x) = x - \frac{1}{4x} \quad \text{So } f'(x)^2 = x^2 - \frac{1}{2} + \frac{1}{16x^2}.$$

$$\begin{aligned} A.L. &= \int_1^4 \sqrt{1 + x^2 - \frac{1}{2} + \frac{1}{16x^2}} dx = \int_1^4 \sqrt{x^2 + \frac{1}{2} + \frac{1}{16x^2}} dx \\ &= \int_1^4 \sqrt{\left(x + \frac{1}{4x}\right)^2} dx = \int_1^4 \left(x + \frac{1}{4x}\right) dx \\ &= \left(\frac{1}{2}x^2 + \frac{1}{4} \ln x\right)_1^4 = 8 + \frac{1}{4} \ln 4 - \frac{1}{2} \end{aligned}$$