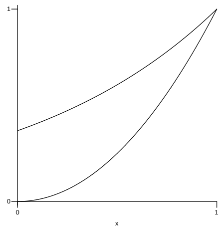
PRACTICE3 Name________________________ Section______

Draw a volume element on each picture then determine the volume of the solid formed when rotating the region bounded by the curves:

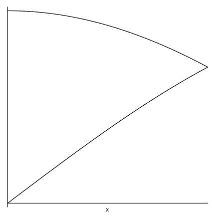
1. $f(x) = e^{x-1}$ and $g(x) = x^2$ about the x-axis. $(0 \le x \le 1)$



$V_{out} = \pi(e^{x-1})^2\Delta x$ and $V_{in} = \pi(x^2)^2\Delta x$

$$\begin{aligned} V &= \int_0^1 \pi\left(e^{2x-2} - x^4\right)\,dx \\ &= \pi\left(\frac{1}{2}e^{2x-2} - \frac{1}{5}x^5\right)_0^1 \\ &= \pi\left(\frac{1}{2} - \frac{1}{5} - \frac{1}{e^2}\right) \end{aligned}$$

2. $f(x) = \cos x$ and $g(x) = \sin x$ about the x-axis. $(0 \le x \le \pi/4)$



$V_{out} = \pi(\cos x)^2\Delta x$ and $V_{in} = \pi(\sin x)^2\Delta x$

$$\begin{aligned} V &= \int_0^{\pi/4} \pi\left(\cos^2 x - \sin^2 x\right)\,dx \\ &= \int_0^{\pi/4} \cos(2x)\,dx \\ &= \pi\left(\frac{1}{2}\sin(2x)\right)_0^{\pi/4} = \frac{\pi}{2} \end{aligned}$$

3. $f(x) = \cos x$ and $g(x) = \sin x$ about the y-axis.

$$
\begin{aligned}
V &= \int_0^{\pi/4} 2\pi x(\cos x - \sin x)\,dx \\
&= 2\pi\left(x(sinx + \cos x) + \cos x - \sin x\right)_0^{\pi/4} \\
&= 2\pi\left(\frac{\pi}{4}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\pi}{4}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - 1\right) \\
&= 2\pi\left(\frac{\sqrt{2}\pi}{4} - 1\right)
\end{aligned}
$$

4. $f(x) = 2x^3 + x$ and $g(x) = 2x + 1$ about the y-axis.

$$
\begin{aligned}
V &= \int_0^1 2\pi x(2x + 1 - (2x^3 + x))\,dx \\
&= \int_0^1 2\pi(x^2 + x - 2x^4)\,dx \\
&= 2\pi\left(\frac{1}{3}x^3 + \frac{1}{2}x^2 - \frac{2}{5}x^5\right)_0^1 \\
&= 2\pi\left(\frac{1}{3} + \frac{1}{2} - \frac{2}{5}\right)
\end{aligned}
$$

5. $f(x) = x^3 - 3x^2 + 3x$ and $g(x) = \frac{1}{8}x^4$ about the y-axis.

$$
\begin{aligned}
V &= \int_0^2 2\pi x\left(x^3 - 3x^2 + 3x - \frac{1}{8}x^4\right)\,dx \\
&= 2\pi\left(\frac{1}{5}x^5 - \frac{3}{4}x^4 + x^3 - \frac{1}{48}x^6\right)_0^2 \\
&= 2\pi\left(\frac{32}{5} - 12 + 8 - \frac{4}{3}\right)
\end{aligned}
$$