PRACTICE2 Name______________________________ Section______

1. Determine the average value of the function $f(x) = \dfrac{4}{1+x^2}$ on $0 \le x \le \sqrt{3}$.
(no decimal answers; evaluate completely)
answer: $AV = \dfrac{1}{\sqrt{3}} \displaystyle\int_0^{\sqrt{3}} \dfrac{4}{1+x^2}\,dx = \dfrac{4}{\sqrt{3}} \arctan x \Big|_0^{\sqrt{3}} = \dfrac{4\pi}{3\sqrt{3}}$

2. Determine the average value of the function $f(x) = \sqrt{x}$ on $1 \le x \le 36$.
answer: $\dfrac{1}{35} \displaystyle\int_1^{36} \sqrt{x}\,dx = \dfrac{2}{105} x^{3/2}\Big|_1^{36} = \dfrac{430}{105}$

3. Determine b so that the average value of $f(x) = 2x + 3$ on $[1, b]$ is 10.
answer: $10 = \dfrac{1}{b-1} \displaystyle\int_1^{10} (2x+3)dx = \left(x^2 + 3x\right)_1^b = \dfrac{b^2 + 3b - 4}{b-1}$
Then
$10b - 10 = b^2 + 3b - 4 \Rightarrow b^2 - 7b + 6 = 0.$
Solve the quadratic equation and $b = 6$.

4. Determine c on $[1, 4]$ so that $f(c)$ is the average value of the function $f(x) = 3x^2 + 2x + 1$ on $[1, 4]$. (Mean Value of the Integral)
answer: first determine average value:
Av$=\dfrac{1}{3} \displaystyle\int_1^4 (3x^2 + 2x + 1)\,dx = \dfrac{1}{3}(x^3 + x^2 + x)\Big|_1^4 = 27$
Then
$3c^2 + 2c + 1 = 27 \Rightarrow 3c^2 + 2c - 26 = 0$
Solve the quadratic equation.

5. Show the evaluation of the integral. Determine the value of the improper integral if it converges. If it diverges, show it.

(a) $\displaystyle\int_0^\infty xe^{-2x^2}\,dx = -\frac{1}{4}e^{-2x^2}\Big|_0^{x=t\to\infty} = \frac{1}{4}.$ Converges

(b) $\displaystyle\int_0^\infty \frac{4x+6}{x^2+3x+7}\,dx = 2\ln\left(x^2+3x+7\right)\Big|_0^{x=t\to\infty} = \infty.$ Diverges

(c) $\displaystyle\int_0^\infty \frac{5}{x^2+9}\,dx = \frac{5}{3}\arctan\left(x/3\right)\Big|_0^{x=t\to\infty} = \frac{5\pi}{6}.$ Converges

(d) $\displaystyle\int_0^1 \frac{4}{\sqrt{1-x^2}}\,dx = 4\arcsin\left(x\right)\Big|_0^{x=t\to 1} = 2\pi.$ Converges

(e) $\displaystyle\int_0^1 \frac{4}{\sqrt{1-x}}\,dx = -2\sqrt{1-x}\Big|_0^{x=t\to 1} = 2.$ Converges

(f) $\displaystyle\int_0^1 \frac{4}{1-x^2}\,dx = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)\Big|_0^{x=t\to 1} = \infty.$ Diverges

(g) $\displaystyle\int_5^{10} \frac{4}{(x-5)^{2/3}}\,dx = 12\sqrt[3]{x-5}\Big|_{x=t\to 5}^{10} = 12\sqrt[3]{5}.$ Converges