Hydrostatic Pressure and Force

Message: Make sure you go through these examples.

Pressure = density × gravity × depth = Weight × Depth.

Force = Pressure × Area. So Force = Weight × Area × Depth

1. An aquarium 5 m long, 10 m wide, and 3 m deep is filled with seawater of density 1030 kg/m³ to a depth of 2.5 m.

   (a) Find the hydrostatic pressure at the bottom of the pool.
       Pressure = 1030 × 9.8 × 2.5.

   (b) Find the hydrostatic force at the bottom of the pool.
       Force = 1030 × 9.8 × 5 × 10 × 2.5.

   (c) Find the hydrostatic force on one end of the aquarium. On 5 × 3 end: Weight at level y m down is 1030 × 9.8. Area at level y m down is 5Δy. Depth at level y m down is y. Therefore, Force_y = 1030 × 9.8 × 5yΔy and

       \[ \text{Force} = \int_0^{2.5} 1030 \cdot 9.8 \cdot 5y \, dy \]

2. The end of a tank containing water is vertical and has the shape of the bottom of a half circle of radius 10 m. Determine the hydrostatic force against the end of the tank.

   Answer: Determine Force = Pressure × Area for a given strip:

   For water we have \( ρg = 1000 \times 9.8 = 9.8 \times 10^3 \).

If we consider the equations of the half circle: \( y = -\sqrt{100 - x^2} \) (negative because it is the bottom half). Then the depth of the water at a value of \( y \) is \( -y \). So that
Pressure = $9.8 \times 10^3 \times (-y)$.
The Area of the strip is twice $x$ coordinate at $y$ so $\text{Area} = 2\sqrt{100-y^2} \Delta y$.
And the Force due to that strip is $F_y = \text{Pressure} \times \text{Area}$.

$$F_y = 9.8 \times 10^3 \times (-y) \times 2\sqrt{100-y^2} \Delta y.$$ 

$$F = 9.8 \times 10^3 \cdot \int_{y=-10}^{y=0} -2y\sqrt{100-y^2} \ dy$$

$$= 9.8 \times 10^3 \cdot \left[ \frac{2}{3} \left(100 - y^2\right)^{3/2} \right]_{y=-10}^{y=0}$$

$$= 9.8 \times 10^3 \times \frac{2}{3} \times 1000 = \frac{2}{3} \times 9.8 \times 10^3.$$ 

3. The end of a tank containing water is vertical and has the shape of a triangle with base 4 ft and height 6 ft. The tank is full to a level of 4 ft. Determine the Force on this end.

**Answer** Determine $F = \text{Pressure} \times \text{Area}$ for a given strip:
For water we have $\rho g = 62.4$.

\[
y = -3x + 6
\]

If we consider the equation of the line with slope -3 and $y$-intercept (0,6), we have $y = -3x + 6$. The depth from the top at any $y$ is $4 - y$. So $\text{Pressure} = 62.4 \times (4 - y)$.

The Area of any strip is twice the $x$-coordinate at $y$ times $\Delta y$, so $\text{Area} = 2 \left( \frac{6-y}{3} \right) \Delta y$.
And the Force due to that strip is $F_y = \text{Force} = \text{Pressure} \times \text{Area}$.

$$F_y = 62.4 \times (4 - y) \times 2 \left( \frac{y - 6}{3} \right) \Delta y.$$
\[ F = \frac{124.8}{3} \int_{y=0}^{y=4} (4 - y)(6 - y) \, dy \]
\[ = \frac{124.8}{3} \int_{y=0}^{y=4} (y^2 - 10y + 24) \, dy \]
\[ = \frac{124.8}{3} \left[ \frac{1}{3}y^3 - 5y^2 + 24y \right]_{y=0}^{y=4} \]

4. A vertical dam has a semicircle gate as shown in the figure. Find the hydrostatic force against the gate.

Determine the Force=Pressure\times Area at any level \( y \) on the gate. The equation of the semicircle is \( y = \sqrt{4 - x^2} \).
The depth of the water at any level \( y \) is \( (10-y) \). So Pressure=\(1000 \times 9.8 \times (10 - y) \).
The Area of any strip is twice the \( x \)-coordinate at \( y \) times \( \Delta y \) so Area=\(2\sqrt{4 - y^2} \Delta y \).
Force=Pressure\times Area

\[ F_y = 9.8 \times 10^3 \times (10 - y) \times 2\sqrt{4 - y^2} \Delta y. \]

\[ F = 19.6 \times 10^3 \cdot \int_{y=0}^{y=2} (10 - y)\sqrt{4 - y^2} \, dy. \]
\[ = 19.6 \times 10^3 \left[ 10 \int_{0}^{2} \sqrt{4 - y^2} \, dy - \int_{0}^{2} y \sqrt{4 - y^2} \, dy \right] \]
\[ = 19.6 \times 10^3 \left[ 10\pi + \frac{2}{3}(4 - y^2)^{3/2} \right]_{0}^{2} \]
\[ = 19.6 \times 10^3 \times \left( 10\pi - \frac{16}{3} \right). \]