Hydrostatic Pressure and Force

Message: Make sure you go through these examples.
Pressure = density×gravity×depth=Weight×Depth.
Force=Pressure×Area. So Force=Weight×Area×Depth

1. An aquarium 5 m long, 10 m wide, and 3 m deep is filled with seawater of density 1030 kg/m^3 to a depth of 2.5 m.

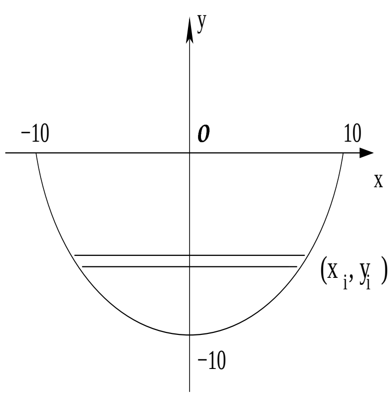
   (a) Find the hydrostatic pressure at the bottom of the pool.
   Pressure=1030×9.8×2.5.

   (b) Find the hydrostatic force at the bottom of the pool.
   Force=1030×9.8×5×10×2.5.

   (c) Find the hydrostatic force on one end of the aquarium. On 5X3 end: Weight at level y m down is 1030×9.8. Area at level y m down is $5\Delta y$. Depth at level y m down is y. Therefore, $\text{Force}_y = 1030 \cdot 9.8 \cdot 5y\Delta y$ and

$$\text{Force} = \int_0^{2.5} 1030 \cdot 9.8 \cdot 5y \, dy$$

2. The end of a tank containing water is vertical and has the shape of the bottom of a half circle of radius 10 m. Determine the hydrostatic force against the end of the tank.
   Answer Determine Force=Pressure×Area for a given strip:
   For water we have $\rho g = 1000 \times 9.8 = 9.8 \times 10^3$.



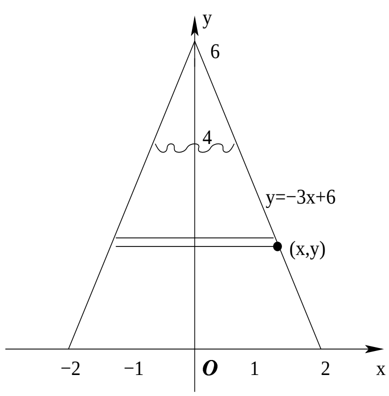
If we consider the equations of the half circle: $y = -\sqrt{100 - x^2}$ (negative because it is the bottom half). Then the depth of the water at a value of y is $-y$. So that

Pressure$=9.8 \times 10^3 \times (-y)$.
The Area of the strip is twice x coordinate at y so Area$=2\sqrt{100-y^2}\Delta y$.
And the Force due to that strip is Force=Pressure×Area.

$$F_y = 9.8 \times 10^3 \times (-y) \times 2\sqrt{100-y^2}\Delta y.$$

$$\begin{aligned} F &= 9.8 \times 10^3 \cdot \int_{y=-10}^{y=0} -2y\sqrt{100-y^2}\, dy \\ &= 9.8 \times 10^3 \cdot \left[\frac{2}{3}\left(100-y^2\right)^{3/2}\right]_{y=-10}^{y=0} \\ &= 9.8 \times 10^3 \times \frac{2}{3}(1000) = \frac{2}{3} \times 9.8 \times 10^3. \end{aligned}$$

3. The end of a tank containing water is vertical and has the shape of a triangle with base 4 ft and height 6 ft. The tank is full to a level of 4 ft. Determine the Force on this end.
   Answer Determine Force=Pressure×Area for a given strip:
   For water we have $\rho g = 62.4$.



If we consider the equation of the line with slope -3 and y-intercept (0,6), we have $y = -3x + 6$. The depth from the top at any y is $4 - y$. So
Pressure$=62.4 \times (4-y)$.
The Area of any strip is twice the x-coordinate at y times Δy, so Area$=2\left(\frac{6-y}{3}\right)\Delta y$.
And the Force due to that strip is Force=Pressure×Area.

$$F_y = 62.4 \times (4-y) \times 2\left(\frac{y-6}{3}\right)\Delta y.$$

$$
\begin{aligned}
F &= \frac{124.8}{3}\int_{y=0}^{y=4}(4-y)(6-y)\,dy\\
&= \frac{124.8}{3}\int_{y=0}^{y=4}(y^2-10y+24)\,dy\\
&= \frac{124.8}{3}\left[\frac{1}{3}y^3-5y^2+24y\right]_{y=0}^{y=4}
\end{aligned}
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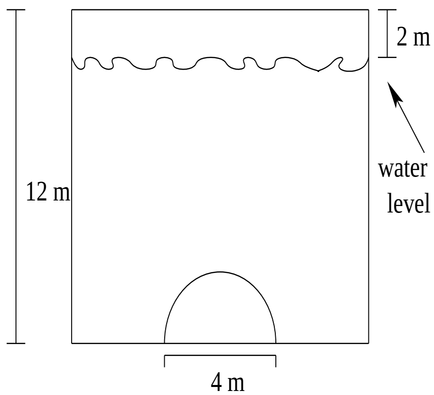
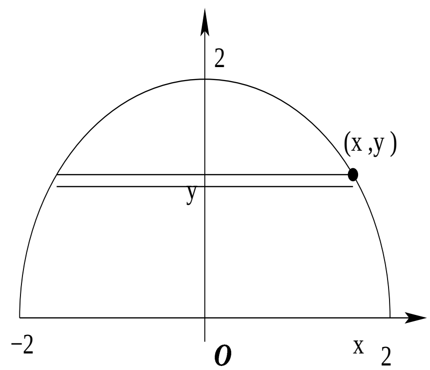
4. A vertical dam has a semicircle gate as shown in the figure. Find the hydrostatic force against the gate.





Determine the Force=Pressure×Area at any level y on the gate. The equation of the semicircle is $y=\sqrt{4-x^2}$.
The depth of the water at any level y is (10-y). So Pressure=$1000\times 9.8\times(10-y)$.
The Area of any strip is twice the x-coordinate at y times Δy so Area=$2\sqrt{4-y^2}\Delta y$.
Force=Pressure×Area

$$F_y = 9.8\times 10^3\times(10-y)\times 2\sqrt{4-y^2}\Delta y.$$

$$
\begin{aligned}
F &= 19.6\times 10^3\cdot\int_{y=0}^{y=2}(10-y)\sqrt{4-y^2}\,dy.\\
&= 19.6\times 10^3\left[10\int_0^2\sqrt{4-y^2}\,dy-\int_0^2 y\sqrt{4-y^2}\,dy\right]\\
&= 19.6\times 10^3\left[10\pi+\frac{2}{3}(4-y^2)^{3/2}\Big|_0^2\right]\\
&= 19.6\times 10^3\times\left(10\pi-\frac{16}{3}\right).
\end{aligned}
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