

Final Exam (Spring-2003)

1. (30points) Find each antiderivative

(a) $\int x e^x dx$

(b) $\int \frac{dx}{(x+2)(x+3)}$

(c) $\int \arcsin x dx$

(d) $\int \sin^3 x dx$

(e) $\int \frac{x^2 + 2x + 1}{x} dx$

2. (16points) Compute the exact value of:

(a) $\int_e^{e^2} \frac{1}{x \ln x} dx$

(b) $\int_{1/2}^{\sqrt{3}/2} \frac{1}{x^2 \sqrt{1-x^2}} dx$

3. (21 points) Evaluate each improper integral that converges. If it is divergent, show why with your derivation.

(a) $\int_1^{\infty} e^{-x/2} dx$

(b) $\int_0^{\pi/2} \sec^2 x \, dx$

(c) $\int_3^4 \frac{1}{\sqrt[3]{x-3}} \, dx$

4. (20 points) Solve each of the following first order initial value problems.

(a) $y' + 2y = e^{-x}, \quad y(0) = 2$

(b) $y' = y^2 \cos x, \quad y(0) = -1/2$

5. (a) (6points) Set up and evaluate the integral for the arc length of the curve
 $x = e^t \sin t, \quad y(t) = e^t \cos t, \quad 0 \leq t \leq \ln 3.$

(b) (6points) Set up but DO NOT evaluate the integral for the length of the graph
 $y = \ln(\sin x), \quad \frac{\pi}{6} \leq x \leq \frac{\pi}{3}.$

6. (40 points) Set up but DO NOT evaluate each definite integral described.

(a) Use disks or washers to find the volume of the solid of revolution formed when the region bounded by $y = x^2$, $y = 0$, $x = 1$ and $x = 3$ is revolved about the x -axis.

(b) Use cylindrical shells to find the volume of the solid of revolution generated when the region between the curve $y = x^2$, above the x -axis and between $x = 0$ and $x = 1$ is revolved about the y -axis.

(c) The work done in pumping the water to the top of a storage tank in the shape of a right cylinder of radius 4 m and height 8 m.

(d) The hydrostatic force on an upright flat plate in the shape of a right triangle with legs 3 ft. and 4 ft. which is immersed in water 5 ft. deep.

7. (15points) Find the radius and interval of convergence for:

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(2x-1)^n}{n}$$

8. (24points) For each series, give the technical reason it converges or diverges by saying what type of series it is or what test applies. Apply the test and state the result. Find the sum exactly, if the series is geometric and convergent.

(a) $\sum_{n=0}^{\infty} \frac{n}{2n+1}$

(b) $\sum_{n=0}^{\infty} \frac{2^{n+1}}{5^n}$

(c) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln n}$

(d) $\sum_{n=1}^{\infty} \frac{n+1}{2n^2+3n}$

9. (a) (5points) Without proof, write the Maclaurin series expansion for $f(x) = x^2 \sin x$
- (b) (5points) State the value of the sum $1 + 2 + \frac{2}{2!} + \frac{2}{3!} + \frac{2}{4!} + \dots$
10. (a) (6points) Using a binomial series, expand $f(x) = \frac{1}{\sqrt[3]{1+x}}$ in a Maclaurin series. Write the series using Σ notation.
- (b) Write the first three Taylor polynomials T_1 , T_2 and T_3 for $f(x)$ above expanded about $a = 0$.