

Outline for Final Exam

1. Integration Techniques

(a) Step One: Look for a u du -substitution

i. $\int \sin x \sqrt{4 - 3 \cos x} \, dx$
 $u = 4 - 3 \cos x \quad \frac{1}{3} du = \sin x \, dx$

ii. $\int \frac{1}{\sqrt{x} + \sqrt[3]{x}} \, dx$
 $u = \sqrt[6]{x} \Rightarrow u^6 = x \quad 6u^5 \, du = dx$
 $u^3 = \sqrt{x} \text{ and } u^2 = \sqrt[3]{x}$

(b) Step Two: Look for a u dV -substitution

i. $\int x \ln x \, dx$
 $u = \ln x \quad dV = x \, dx$

ii. $\int \sin^{-1} x \, dx \quad u = \sin^{-1} x \quad dV = dx$

(c) Step Three: Use Trig Identities (Refer to review sheet)

Message: You must be familiar with identities!

(d) Step Four: Look for Trig Substitution

i. $\int \frac{x^2}{\sqrt{1 - 4x^2}} \, dx$. Set up the triangle.
 $\sin \theta = 2x$ so that $dx = \frac{1}{2} \cos \theta$ and
 $\sqrt{1 - 4x^2} = \cos \theta$ and $x^2 = \frac{1}{4} \sin^2 \theta$

(e) Step Five: Look for Partial Fractions

i. $\int \frac{3}{x^2 - 3x + 2} = \int \left(\frac{3}{x - 2} - \frac{3}{x - 1} \right) dx$

2. Applications of Integration

(a) Area between Curves

(b) Volume

i. Disk Method

$$\int_a^b \pi (f(x))^2 \, dx$$

ii. Washer Method

$$\int_a^b \pi (f(x)^2 - g(x)^2) \, dx$$

iii. Shell Method

$$\int_a^b 2\pi x \cdot f(x) dx$$

(c) Arc Length

i. Parametric Form

$$\int_{t=a}^{t=b} \sqrt{(dx/dt)^2 + (dy/dt)^2} dt$$

ii. Function Form

$$\int_{x=a}^{x=b} \sqrt{1 + (dy/dx)^2} dx$$

(d) Center of Mass

$$\bar{x} = \frac{\int_a^b x \cdot f(x) dx}{\int_a^b f(x) dx} \quad \bar{y} = \frac{\int_a^b \frac{1}{2}(f(x))^2 dx}{\int_a^b f(x) dx}$$

(e) Work (See Review for exam 1)

Work=Force*Distance

To empty the water out of the top of a full parabolic bowl having sides the equation $y = x^2$ for $0 \leq x \leq 1$ the

Force at any level y is the weight/ft³ times the volume so $F = 62.4 \cdot \pi(\sqrt{y})^2 \Delta y$

Distance is the distance y is from the top so $D = 1 - y$. And

$$W = 62.4\pi \int_0^1 y(1 - y) dy$$

(f) Hydrostatic Pressure (See Hydrostatic-Pressure-Problems)

Message: Make sure you go through these examples.

Pressure = density*gravity*depth=Weight*Depth.

Force=Pressure*Area. So Force=Weight*Area*Depth

i. An aquarium 5 m long, 10 m wide, and 3 m deep is filled with seawater of density 1030 kg/m³ to a depth of 2.5 m.

A. Find the hydrostatic pressure at the bottom of the pool.

$$\text{Pressure}=1030*9.8*2.5.$$

B. Find the hydrostatic force at the bottom of the pool.

$$\text{Force}=1030*9.8*5*10*2.5$$

C. Find the hydrostatic force on one end of the aquarium. On 5X3 end:

Weight at level y m down is $1030*9.8$. Area at level y m down is $5\Delta y$.

Depth at level y m down is y . Therefore, $\text{Force}_y = 1030 \cdot 9.8 \cdot 5y\Delta y$

and

$$\text{Force} = \int_0^{2.5} 1030 \cdot 9.8 \cdot 5y dy$$

3. Differential Equations

(a) Separable

Separate the equations so that you achieve $f(y) dy = g(x) dx$

i. Mixing Problems

Draw the picture and label the units of rate-in and rate-out.

$$\frac{dy}{dt} = \text{rate-in} - \text{rate-out}$$

(b) First-Order Linear

First put in the form $y' + p(x)y = q(x)$. Then get the integrating factor which is $I(x) = e^{\int p(x) dx}$. Then

$$I(x)y = \int q(x)I(x) dx$$

4. Sequences and Series

(a) Sum of a Geometric Series: $\sum_{n=k}^{\infty} ar^n = \frac{ar^k}{1-r}$.

(b) Tests for Convergence

i. Integral Test

A. $\sum_{n=0}^{\infty} \frac{n^2}{n^3+1}$ diverges since $\int_0^{\infty} \frac{x^2}{x^3+1} dx = \frac{1}{3} \ln(x^3+1) \Big|_{x=0}^{x \rightarrow \infty}$ diverges.

B. $\sum_{n=0}^{\infty} \frac{1}{n^2+1}$ converges since $\int_0^{\infty} \frac{1}{x^2+1} dx = \arctan x \Big|_{x=0}^{x \rightarrow \infty}$ converges.

ii. Alternating Test

A. $\sum_{n=0}^{\infty} \frac{(-1)^n(n+1)}{2n+3}$ diverges. Even though it is alternating the terms approach $1/2$ not zero.

B. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+3}$ converges with this series alternating and the terms going to zero.

iii. Ratio Test

Take $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$ and if this limit is less than 1, the series converges, if not the series diverges.

(c) Power Series

- i. For functions that can be written in the form $f(x) = \frac{g(x)}{1 - \heartsuit}$ use the geometric expansion:

$$f(x) = \frac{g(x)}{1 - \heartsuit} = g(x) \sum_{n=0}^{\infty} \heartsuit^n.$$

Then simplify by extending $g(x)$ into the sum.

- ii. For the general functions: be familiar with the list on page 630.

Message: You may wish to memorize these if possible.

- iii. For Power Series about $x = a$, set up

$$f(x) = c_0 + c_1(x - a) + c_2(x - a)^2 + c_3(x - a)^4 + c_4(x - a)^5 + \dots$$

and fill in the c 's by making the n th derivative of $f(x)$ on the left equal the n th derivative of the polynomial on the right at $x = a$.

In the end, you'll note that $c_n = \frac{f^n(a)}{n!}$ so that

$$f(x) = \sum_{n=0}^{\infty} c_n(x - a)^n = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!} (x - a)^n.$$

- iv. For functions that can be written in the form $f(x) = a(1 + \heartsuit)^\diamond$ use the binomial expansion:

$$f(x) = a(1 + \heartsuit)^\diamond = a \sum_{n=0}^{\infty} \binom{\diamond}{n} \heartsuit^n.$$

Then simplify by extending a into the sum and if time permits write out two or three terms.