Practice Final I for Calculus II

1. Evaluate each of the integrals

   (a) $\displaystyle\int \frac{x + 2\sin(4x)}{x^2 - \cos(4x)}\,dx$

   (b) $\displaystyle\int \frac{dx}{x^{1/3} - x}\,dx$ hint: let $u = x^{1/3}$

   (c) $\displaystyle\int x^2 \ln x\,dx$

   (d) $\displaystyle\int (x^2 + 100)\sqrt{x - 7}\,dx$

2. Find the area of the planar region bounded by $y = x + 5$, $y = -1$, $y = 2$, $y^2 = x$

3. Find the volume generated by rotating the planar region enclosed by $y = \ln x$, the x axes, and the line $x = 2$.

   (a) about the x axis

   (b) about the y axis

4. Find the length of the curve $x = 3y^{3/2} - 1$ from $y = 0$ to $y = 4$.

5. Find the average value of the function $y = e^x - \dfrac{1}{x}$ over $1 \leq x \leq 3$.

6. A certain virus is known to spread among a population at a rate proportional to the product of the fraction y of the population that has been infected dwith the virus and the fraction that has not yet been infected.

   (a) Write a differential equation that is satisfied by y.

   (b) Solve the differential equation.

   (c) In a small town of 4000 people, at 12 p.m. noon, 100 people have been infected by the virus. By 6 p.m., half of the population had been infected. How many people were infected by 10 p.m.?

7. Evaluate

   (a) $\displaystyle\lim_{n\to\infty} \frac{2^n}{3^n - 5}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{3^{n+1}}{7^n}$

8. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$ converge or diverge? Completely justify your answer.

9. Find the Taylor polynomial of degree 3 for $f(x) = \ln x$ centered at $x = 2$.

10. Suppose $f(x) = e^{-x}$

(a) Find the Maclaurin series for $f(x)$.

(b) Evaluate $\int_0^1 e^{-x}\, dx$ using series. Leave your answer in series form.

11. Use power series to solve the differential equation with initial value $y'' = y$, $y(0) = 1$, $y'(0) = 1$.

ANSWERS

1. (a) $\ln |x^2 - \cos(4x)| + c$

   (b) $\dfrac{-3}{2} \ln |1 - x^{1/3}| + c$

   (c) $\dfrac{x^3}{3} \ln x - \dfrac{x^3}{6} + c$

   (d) $\dfrac{2}{7}(x-7)^{7/2} + \dfrac{28}{5}(x-7)^{5/2} + \dfrac{198}{3}(x-7)^{3/2}$

2. $\displaystyle\int_{-1}^{2} (y^2 - x + 5)\, dx \;=\; 4.5$

3. (a) $\displaystyle\int_{-1}^{2} \pi(\ln x)^2\, dx$

   (b) $\displaystyle\int_{-1}^{2} 2\pi x \ln x\, dx$

4. $\displaystyle\int_0^4 \sqrt{(\frac{9}{2} y^{1/2})^2 + 1}\, dx$

5. $\displaystyle\frac{1}{2}\int_1^3 (e^x - \frac{1}{x})\, dx$

6. (a) $\dfrac{dy}{dt} = ky(1-y)$

   (b) $y(t) = \dfrac{1}{1 + Ae^{-kt}}$

   (c) $y(t) = \dfrac{1}{1 + 39e^{t(\ln 39)/6}}$. Evaluate t=10.

7. (a) $\frac{2}{3}$

   (b) $\frac{27/49}{1-3/7} = \frac{27}{28}$

8. converges since alternating and the terms b_n go to zero.

9. $f^0(2) = \ln 2$ $f^1(2) = \frac{1}{2}$ $f^2(2) = \frac{-1}{2^2}$ $f^3(2) = \frac{2}{2^3}$ $f^4(2) = \frac{-6}{2^4}$
   So $T_4 = \ln 2 + \frac{1}{2}(x-2) - \frac{1}{8}(x-2)^2 + \frac{1}{24}(x-2)^3 - \frac{1}{64}(x-2)^4$

10. (a) $e^{-x} = \displaystyle\sum_{n=0}^{\infty} \frac{(-x)^n}{n!}$

(b) $\int_0^1 e^{-x}\,dx = \sum_{n=1}^{infty} \frac{(-1)^{n+1}}{n!}$

11. $y = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$ which is $y = e^x$

Practice Final II for Calculus II

1. Evaluate the following integrals:
   (a) $\int \cos t (1 + \sin t)^{28}\, dt$
   (b) $\int_1^a \ln x\, dx$
   (c) $\int_1^\infty \frac{1}{2z} dz$
   (d) $\int_{-5}^5 \frac{t}{2+\cos t} dt$

2. Determine whether the following converge or diverge:
   (a) $\{\frac{n+1}{e^n}\}$
   (b) $\sum_{n=0}^\infty \frac{10^n}{n!}$
   (c) $\sum_{n=1}^\infty \frac{n!}{n^n}$
   (d) $\sum_{n=1}^\infty \frac{3}{\sqrt{n}}$

3. Find the interval and radius of convergence for the power series: $\sum_{n=1}^\infty \frac{x^n}{n^3 3^n}$

4. Use power series to solve the initial value problem (find the first 5 nonzero terms of the series) $y' = y + \frac{1}{1+x}, y(0) = 1$

5. A bowl in the shape of a parabola $y = x^2$ for $0 \le x \le 5$ is formed by rotating the curve about the y-axis.
   (a) Find the volume V of the bowl.
   (b) Find the work W done to pump out through the top a quantity of water filling the bowl to a height of 12 ft. Use 62.4 lbs as the weight of 1 ft^3 of water.

6. Solve each of the following inital value problems:
   (a) $\frac{dy}{dx} = -2y + 1$ with $y(0) = 1$
   (b) $\frac{dz}{dt} = 2tz^2$ with $z(0) = 1$

7. Use the exponential and geometric series expansions to evaluate teh following sums:
   (a) $\sum_{n=0}^\infty \frac{2^n}{n!}$
   (b) $\sum_{n=0}^\infty \frac{2^n+3n}{n!}$
   (c) $\sum_{n=0}^\infty \frac{2^n}{5^n}$
   (d) $\sum_{n=0}^\infty \frac{2^{2n+3}}{3^{3n+1}}$

8. Determine the fourth degree Taylor polynomial representing $f(x) = e^{2x}\sin x$ at $x = 0$.

9. Decide with reason which of the following improper integrals is convergent:

   (a) $\int_2^\infty \frac{1}{x^2}dx$

   (b) $\int_2^\infty \frac{1}{x}dx$

   (c) $\int_2^\infty \frac{1}{x\ln x}dx$

   (d) $\int_2^\infty \frac{1}{x(\ln x)^2}dx$

10. Evaluate (show all work)

    (a) $\int_2^5 \frac{1}{x\ln x}dx$

    (b) $\int_2^5 \frac{1}{x(\ln x)^2}$

    (c) $\int_{-\infty}^\infty \frac{dx}{1+x^2}dx$

    (d) $\int_0^{\pi/3} \sin x\sqrt{4+10\cos x}dx$

ANSWERS

1. (a) $\frac{1}{29}(1+\sin t)^{29}$

   (b) $a\ln a - a + 1$

   (c) Diverges

   (d) 0 ODD FUNCTION

2. (a) Converges (limit of the terms is 0)

   (b) Converges (sum is e^{10})

   (c) Converges (ratio test yields 1/e)

   (d) Diverges

3. Interval $-3 \leq x \leq 3$. radius is 3

4. $y = 1 + 2x + \frac{1}{2}x^2 + \frac{1}{2}x^3 - \frac{1}{8}x^4$

5. (a) $\frac{25\pi}{2}$

   (b) $62.4\pi \displaystyle\int_0^{12} (25y - y^2)\,dy$

6. (a) $y(t) = \frac{1}{2} + \frac{1}{2}e^{-2t}$

   (b) $z(t) = \frac{-1}{t^2-1}$

7. e^2 $\quad$ $e^2 + e^3$ $\quad$ $5/3$ $\quad$ $72/23$

8. $T_4(x) = x + 2x^2 + \frac{11}{6}x^3 + x^4$

9. (a) converges (integral test or p-test: p=2)

(b) diverges (integral test, harmonic series)

(c) diverges (integral test, $\int \frac{1}{u}\, du$)

(d) converges (integral test, $\int \frac{1}{u^2}\, du$)

10. (a) $\ln(\ln 5) - \ln(\ln 2)$

(b) $\frac{1}{\ln 5} - \frac{1}{\ln 2}$

(c) π

(d) $\frac{-1}{10}(9^{3/2} - 14^{3/2})$